

Process Mining

Part II – Workflow discovery algorithms

Induction of Control-Flow Graphs

α -algorithm

Heuristic Miner

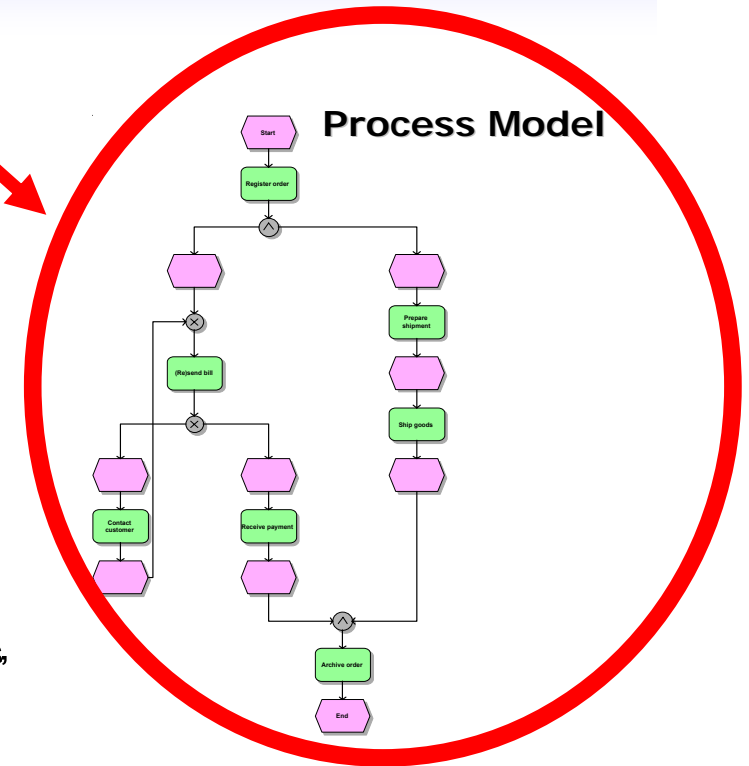
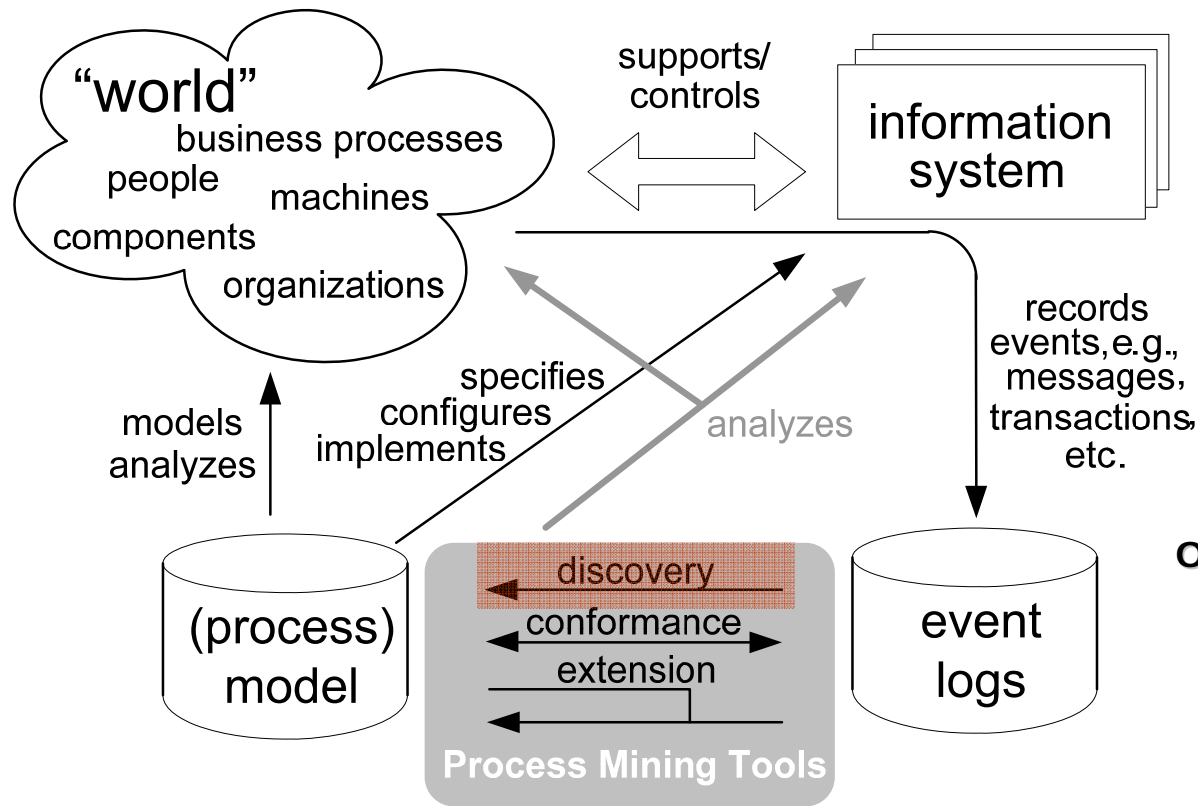
Fuzzy Miner



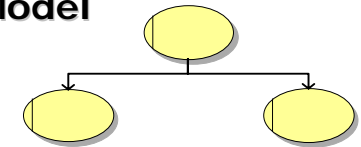
Outline

- Part I – Introduction to Process Mining
 - Context, motivation and goal
 - General characteristics of the analyzed processes and logs
 - Classification of Process Mining approaches
- Part II – Workflow discovery
 - Induction of basic Control Flow graphs
 - Other approaches (α -algorithm, Heuristic Miner, Fuzzy mining)
- Part III – Beyond control-flow mining
 - Organizational mining
 - Social net mining
 - Extension algorithms
- Part IV – Evaluation and validation of discovered models
 - Conformance Check
 - Log-based property verification
- Part V – Clustering-based Process Mining
 - Discovery of hierarchical workflow models
 - Discovery of process taxonomies
 - Outlier detection

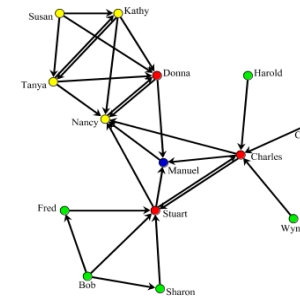
Control-flow discovery



Organizational Model

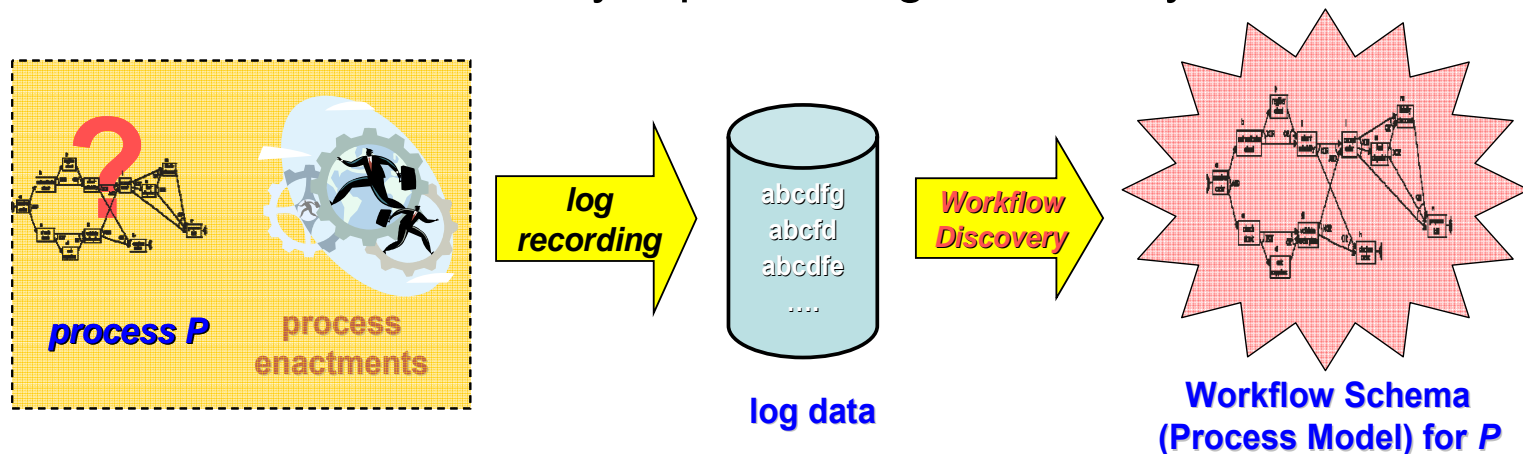


Social Network



Workflow (control flow) discovery

- Input: execution data of a process P (possibly unknown)
 - log: a list of traces
 - In the simplest case each trace just registers the sequence of tasks performed during one execution of P
- Output: a schema for process P
 - captures the P 's behavior, by representing all the ways its tasks are executed



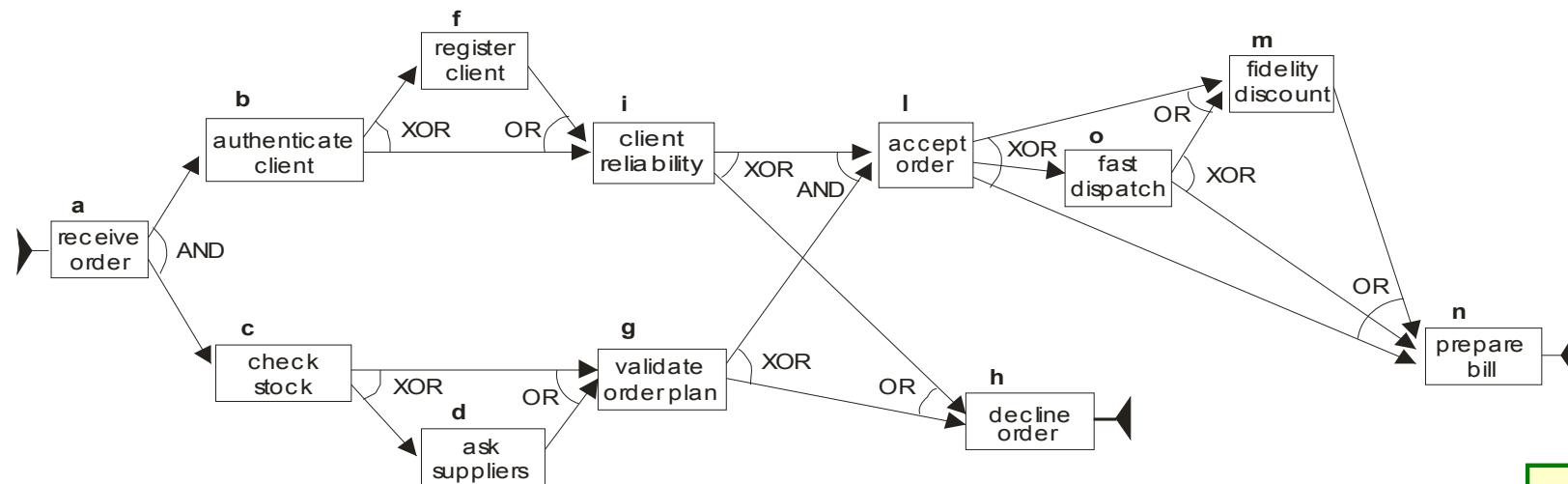
- Usefulness of mined models
 - Help better comprehend process behavior
 - Support process (re)-design (What is the process?)
 - Delta analysis (Are we doing what was specified?)
 - Process Design is often a complex and time consuming task
 - Sometimes, a fully-specified model is not available for the process

Representation of mined models

- A plethora of meta-models for representing workflow models
 - Block-structured languages, Petri Nets, Logics, Process Algebra,...
 - Graph-based languages are a reasonable choice w.r.t. expressiveness, complexity and comprehensibility
 - Most approaches derive some kind of graph over the tasks
 - Few exceptions use alternative techniques (e.g., grammar induction, term rewriting)
- A simple formalism: Control Flow Graph
 - Intuitively specifies which execution flows are allowed across the tasks
 - A labeled, directed graph
 - Each node corresponds to a task (and vice-versa)
 - Each arc represents a (temporal) precedence between two tasks
 - Cardinality constraints further (locally) restricts the possible execution flows

Control Flow Graph (CFG) models

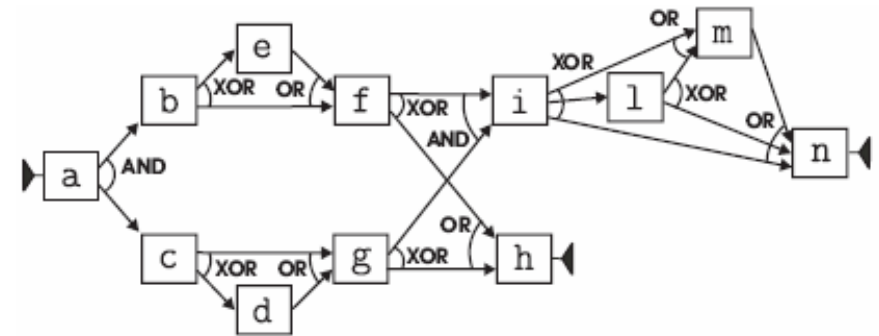
- A CFG schema W for P is a tuple $\langle A, E, a_0, A_F, Fork, Join \rangle$ where:
 - A is a finite set of *activities* (also nodes or tasks);
 - $E \subseteq (A - A_F) \times (A - \{a_0\})$ is an acyclic relation of precedence among activities;
 - $a_0 \in A$ is the starting activity, $A_F \subseteq A$ is the set of final activities;
 - Local constraints are expressed through the functions
 - **Fork:** $(A - A_F) \alpha \{AND, OR, XOR\}$ and
 - **Join:** $(A - \{a_0\}) \alpha \{AND, OR\}$
- Example: a CFG for the toy process *Order Management*



abfcigln,
acbidpegln
abfcdgh

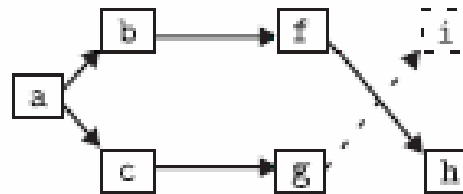
CFG models: executions

schema W



■ Instance of W :

- Connected sub-graph of S 's CFG, containing at least the starting activity and one final activity, compliant with the constraints



■ Trace of the process P :

- A sequence of P 's tasks

■ A trace s is **compliant** with the schema W if there is at least an instance lw of W such that s is a topological order of lw

- Es: the trace $abfcgh$ is compliant with the instance, while the traces $afbcgh$ and $afblm$ are not

Conformance of a CFG schema w.r.t. a log

Two criteria to compare a (mined) model W with a given log L :

- **Completeness:**

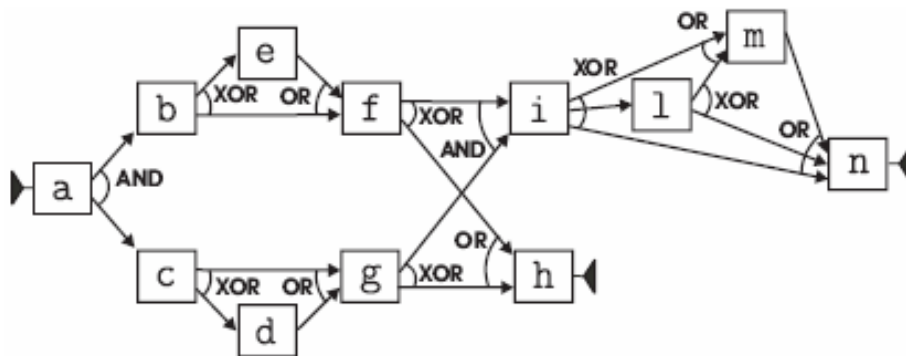
- the percentage of traces in the log that are compliant with W – the larger the more complete

- **Soundness:**

- the percentage of traces that can be generated from W that actually occur in the log – the larger the sounder.

CFG conformance: Example

Schema W



Log L

s_1 : acbgfh	s_9 : abefcgin
s_2 : abfcgh	s_{10} : acgbefiln
s_3 : acgbfh	s_{11} : abcedfgin
s_4 : abcgfin	s_{12} : acdbefgin
s_5 : abfcgimn	s_{13} : abcf dgimn
s_6 : acbfgiln	s_{14} : acdbfgimn
s_7 : acbgfilmn	s_{15} : abcdgfimn
s_8 : abcegfiln	s_{16} : acbfdgin

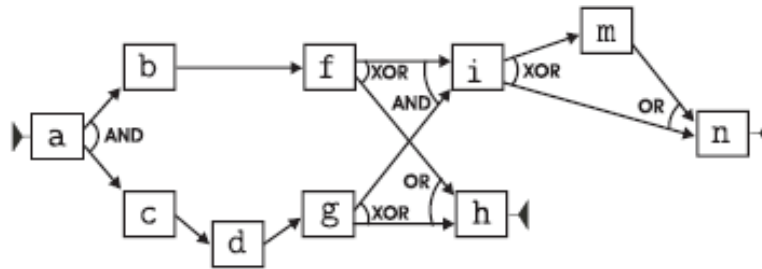
Admitted Instances = 20;

Modeled Traces = 276.

$$\text{soundness}(\{W, L\}) = 16/276 \\ = 5.797\%$$

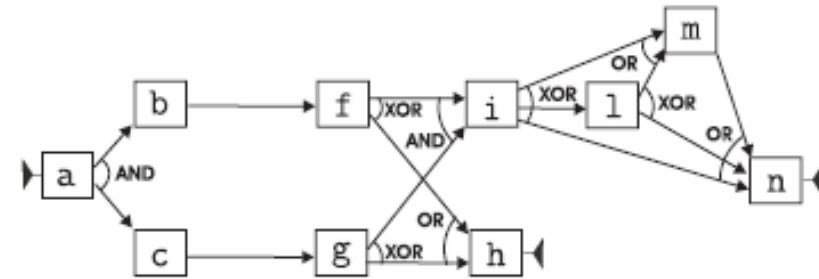
$$\text{completeness}(\{W, L\}) = 16/16 \\ = 100\%$$

Example: a way to get higher soundness



W_1

Modeled Traces = 64



W_2

Modeled Traces = 33

Considered trace Log (L)

s_1 : acbgfh	s_9 : abefcgin
s_2 : abfcgh	s_{10} : acgbefiln
s_3 : acgbfh	s_{11} : abcedfgin
s_4 : abcgfin	s_{12} : acdbefgin
s_5 : abfcgimn	s_{13} : abcfdbgimn
s_6 : acbfgiln	s_{14} : acdbfgimn
s_7 : acbgfilmn	s_{15} : abcdgfinn
s_8 : abcegfiln	s_{16} : acbfdgin

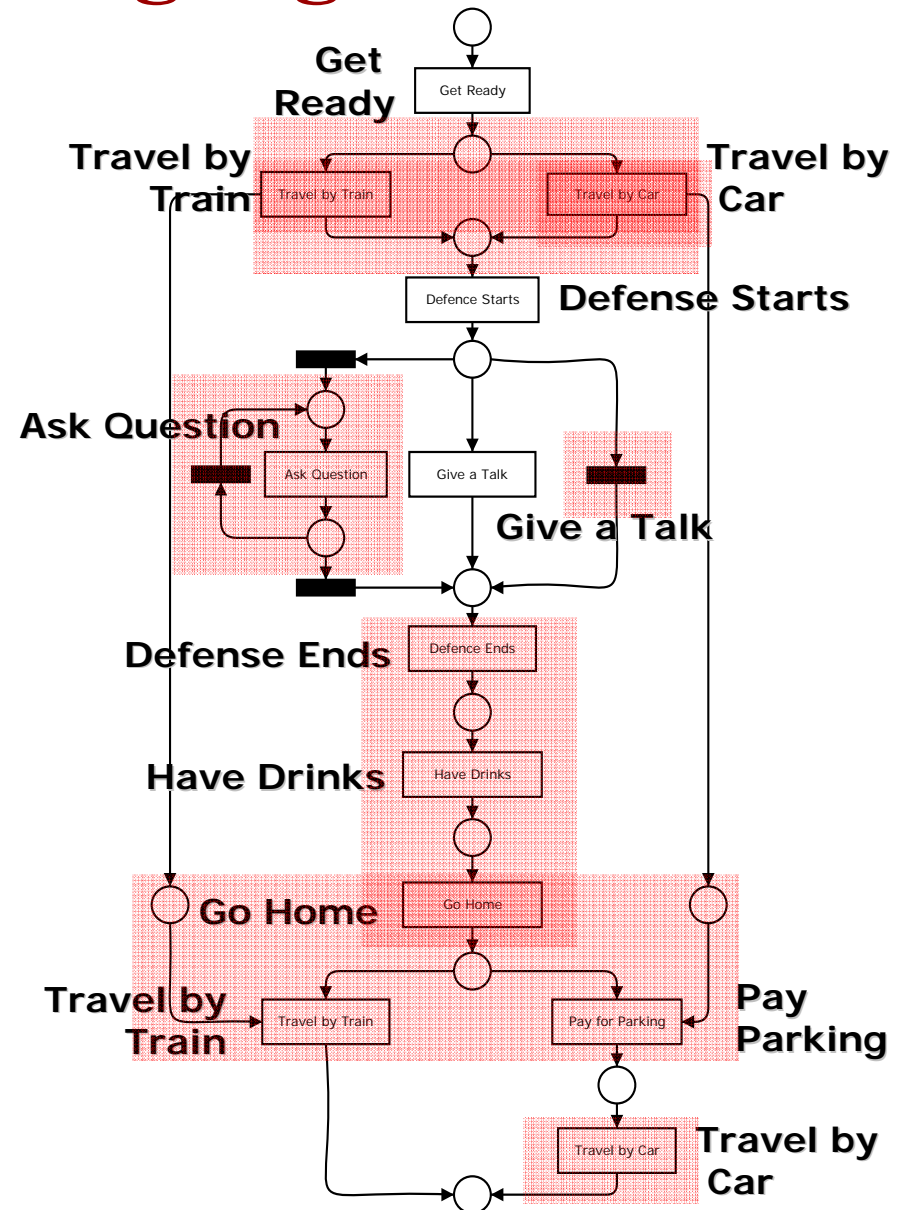
$s_{8,...,12}$ comply with $W_1 \cup W_2$

$soundness(W_1 \cup W_2, L) = 11/97 = 11.34\%$

$completeness(W_1 \cup W_2, L) = 11/16 = 68.75\%$

Other representation languages: Petri nets

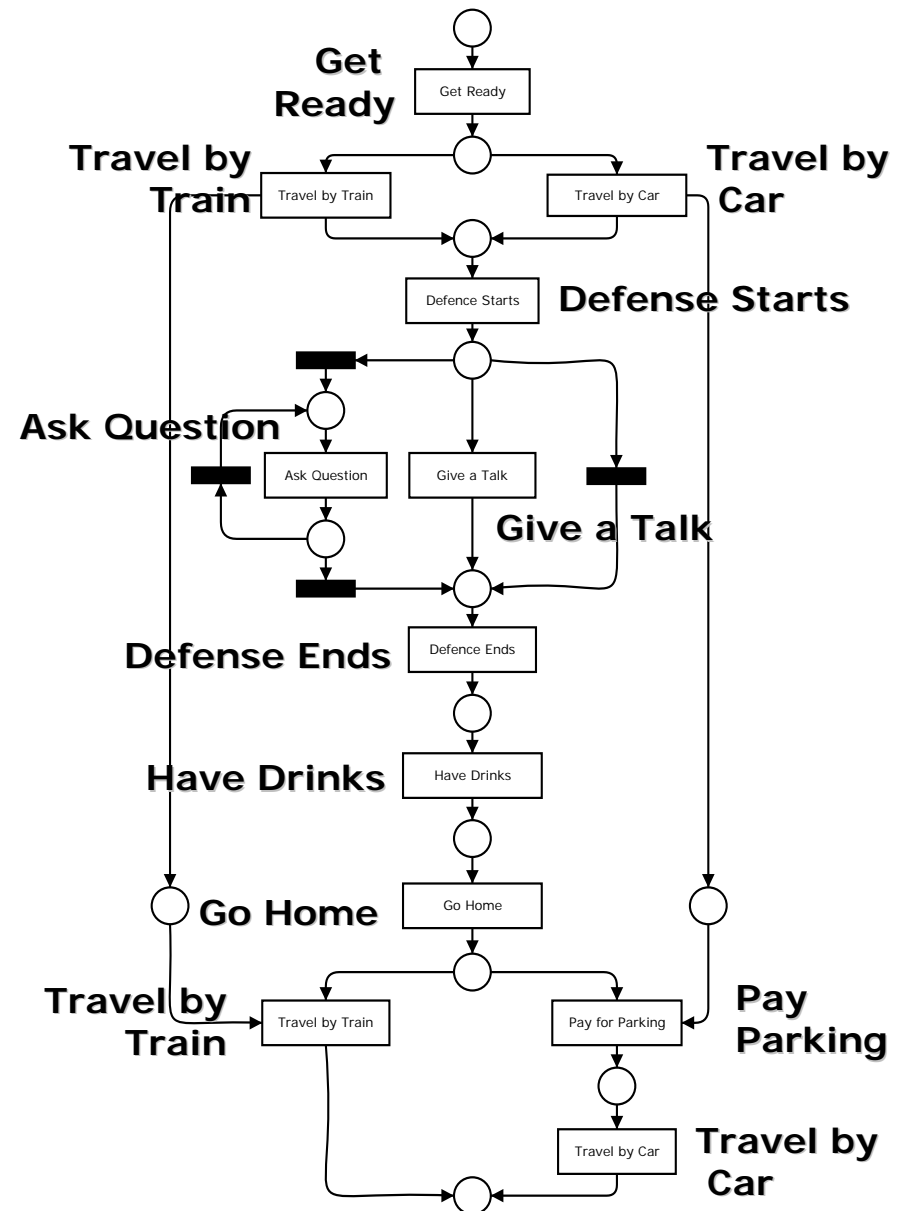
- Sequence
- Splits
- Joins
- Loops
- Non-Free Choice
- Invisible Tasks
- Duplicate Tasks



Other representation languages: Petri nets

- Sequence
- Splits
- Joins
- Loops
- Non-Free Choice
- Invisible Tasks
- Duplicate Tasks

+ noise in logs!



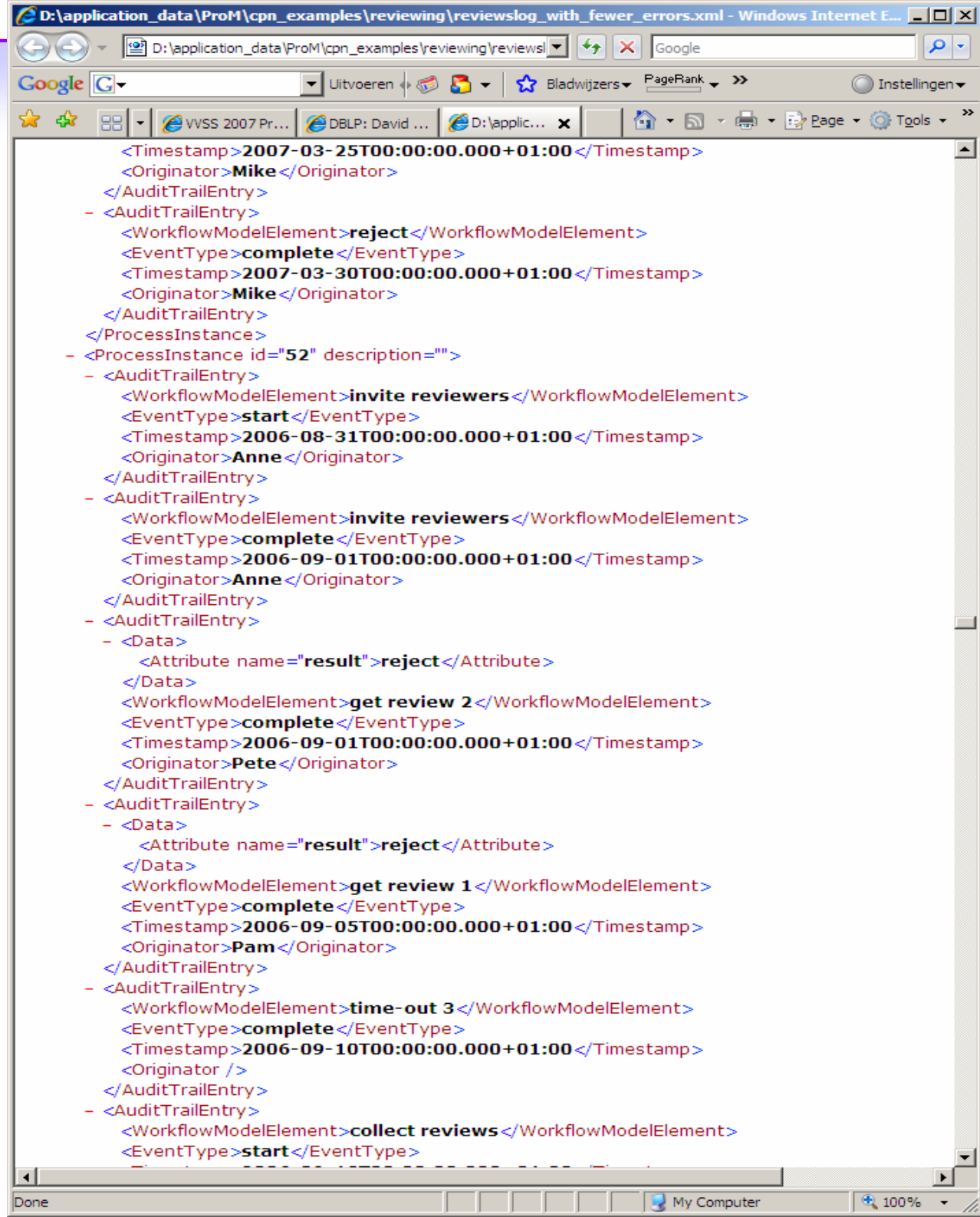
Toy example: paper reviewing

Event log:

- processes
 - process instances
 - events

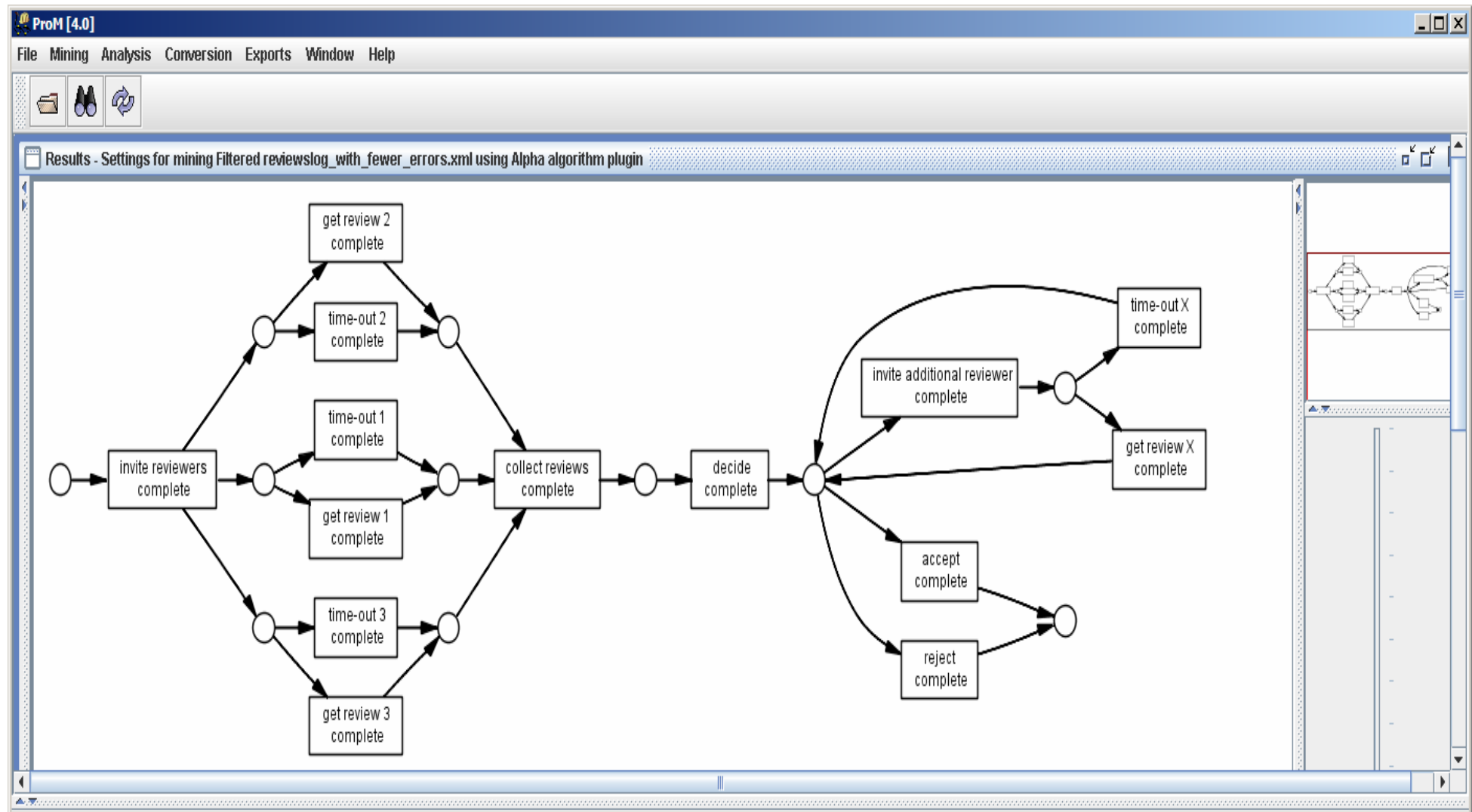
Per event:

- activity name
- (event type)
- (originator)
- (timestamp)
- (data)



```
<Timestamp>2007-03-25T00:00:00.000+01:00</Timestamp>
<Originator>Mike</Originator>
</AuditTrailEntry>
- <AuditTrailEntry>
  <WorkflowModelElement>reject</WorkflowModelElement>
  <EventType>complete</EventType>
  <Timestamp>2007-03-30T00:00:00.000+01:00</Timestamp>
  <Originator>Mike</Originator>
</AuditTrailEntry>
</ProcessInstance>
- <ProcessInstance id="52" description="">
  - <AuditTrailEntry>
    <WorkflowModelElement>invite reviewers</WorkflowModelElement>
    <EventType>start</EventType>
    <Timestamp>2006-08-31T00:00:00.000+01:00</Timestamp>
    <Originator>Anne</Originator>
  </AuditTrailEntry>
  - <AuditTrailEntry>
    <WorkflowModelElement>invite reviewers</WorkflowModelElement>
    <EventType>complete</EventType>
    <Timestamp>2006-09-01T00:00:00.000+01:00</Timestamp>
    <Originator>Anne</Originator>
  </AuditTrailEntry>
  - <AuditTrailEntry>
    <Data>
      <Attribute name="result">reject</Attribute>
    </Data>
    <WorkflowModelElement>get review 2</WorkflowModelElement>
    <EventType>complete</EventType>
    <Timestamp>2006-09-01T00:00:00.000+01:00</Timestamp>
    <Originator>Pete</Originator>
  </AuditTrailEntry>
  - <AuditTrailEntry>
    <Data>
      <Attribute name="result">reject</Attribute>
    </Data>
    <WorkflowModelElement>get review 1</WorkflowModelElement>
    <EventType>complete</EventType>
    <Timestamp>2006-09-05T00:00:00.000+01:00</Timestamp>
    <Originator>Pam</Originator>
  </AuditTrailEntry>
  - <AuditTrailEntry>
    <WorkflowModelElement>time-out 3</WorkflowModelElement>
    <EventType>complete</EventType>
    <Timestamp>2006-09-10T00:00:00.000+01:00</Timestamp>
    <Originator />
  </AuditTrailEntry>
  - <AuditTrailEntry>
    <WorkflowModelElement>collect reviews</WorkflowModelElement>
    <EventType>start</EventType>
```

A discovered Petri net model (α -algorithm)

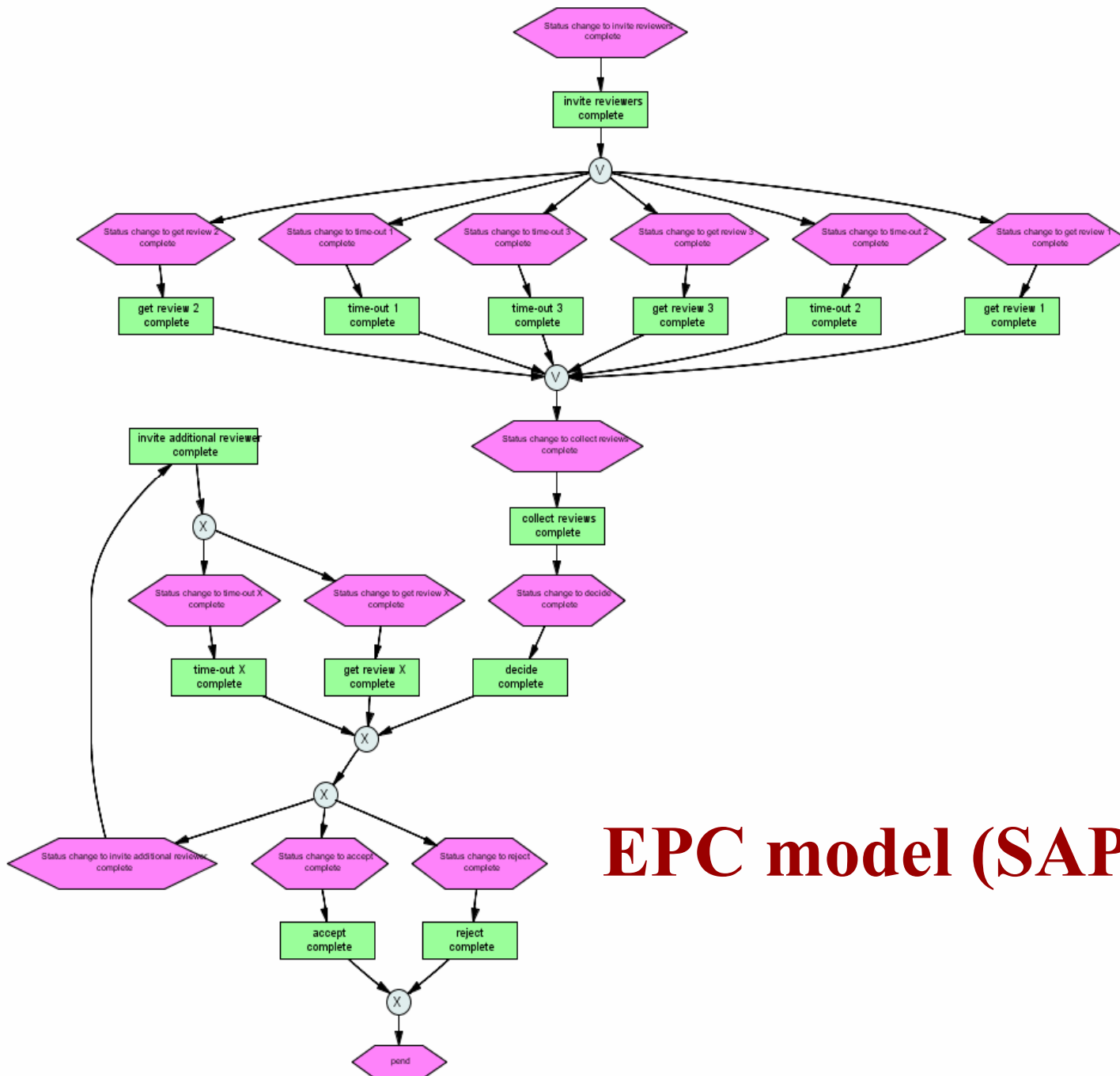


Other workflow languages: EPCs

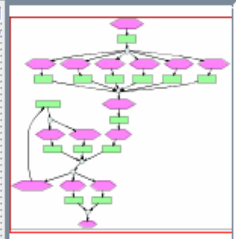
- EPC= Event Driven Process Chain
- An EPC consists of three kinds of elements, which define the flow of a business process as a chain of events.
 - **Functions:** A function corresponds to an activity (task, process step) which needs to be executed.
 - **Events:** Events describe the situation before and/or after a function is executed.
 - **Connectors:** There are three types of connectors: ^ (and), X (xor) and V (or).
- Functions, events and connectors can be connected with edges in such a way that the following rules apply:
 - Events have at most one incoming edge and at most one outgoing edge.
 - Functions have precisely one incoming edge and precisely one outgoing edge.
 - Connectors have either one incoming edge and multiple outgoing edges, or multiple incoming edges and one outgoing edge.
 - In every path, functions and events alternate.
 - No two functions are connected and no two events are connected, not even when there are connectors in between.



Conversion - Labeled WF net to EPC

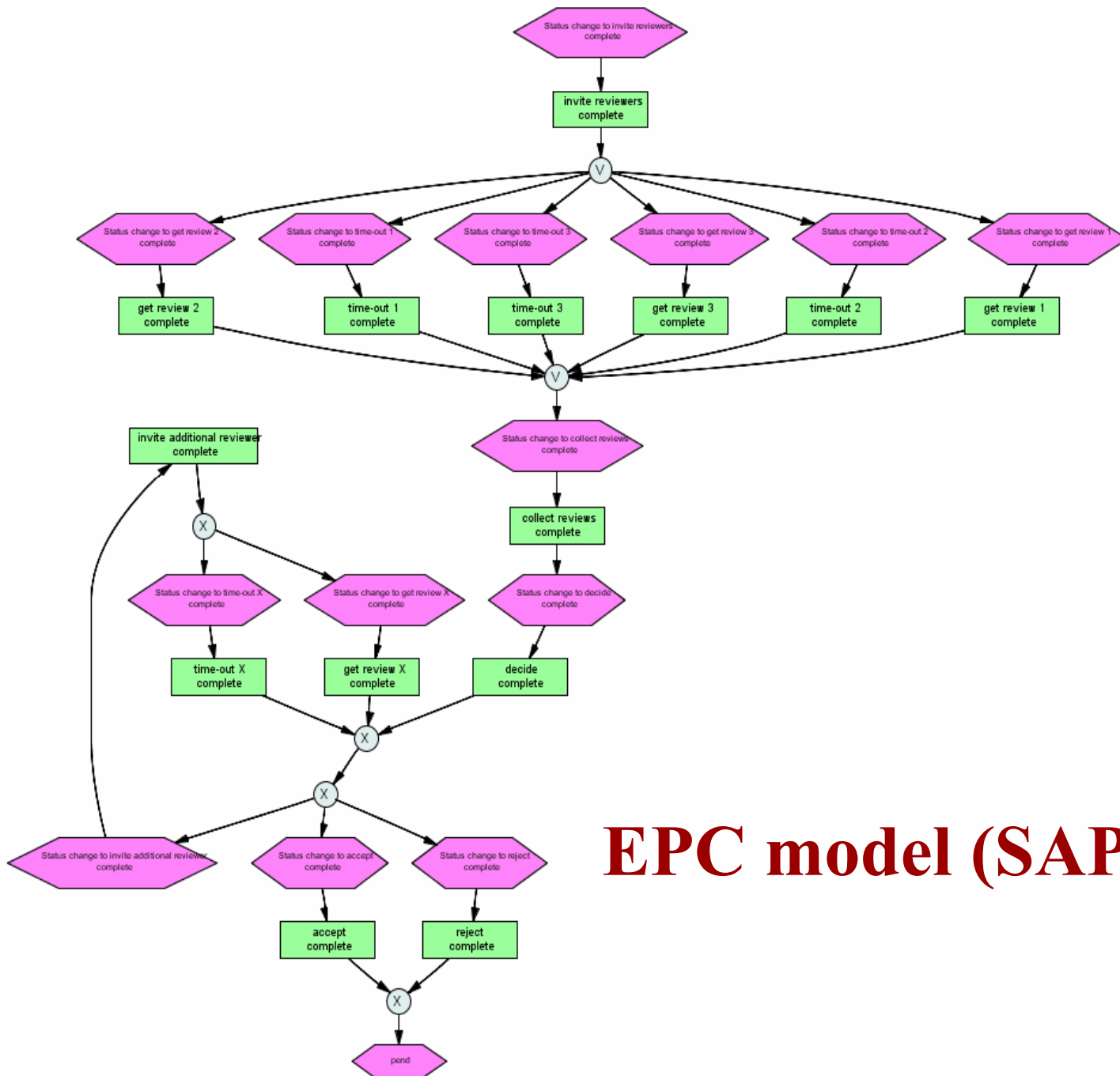


EPC model (SAP, ARIS, etc)

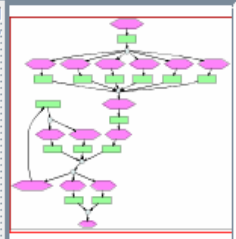




Conversion - Labeled WF net to EPC



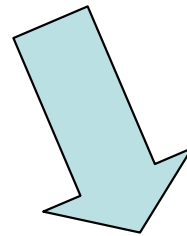
EPC model (SAP, ARIS, etc)



Workflow discovery algorithms: the case of CFG models

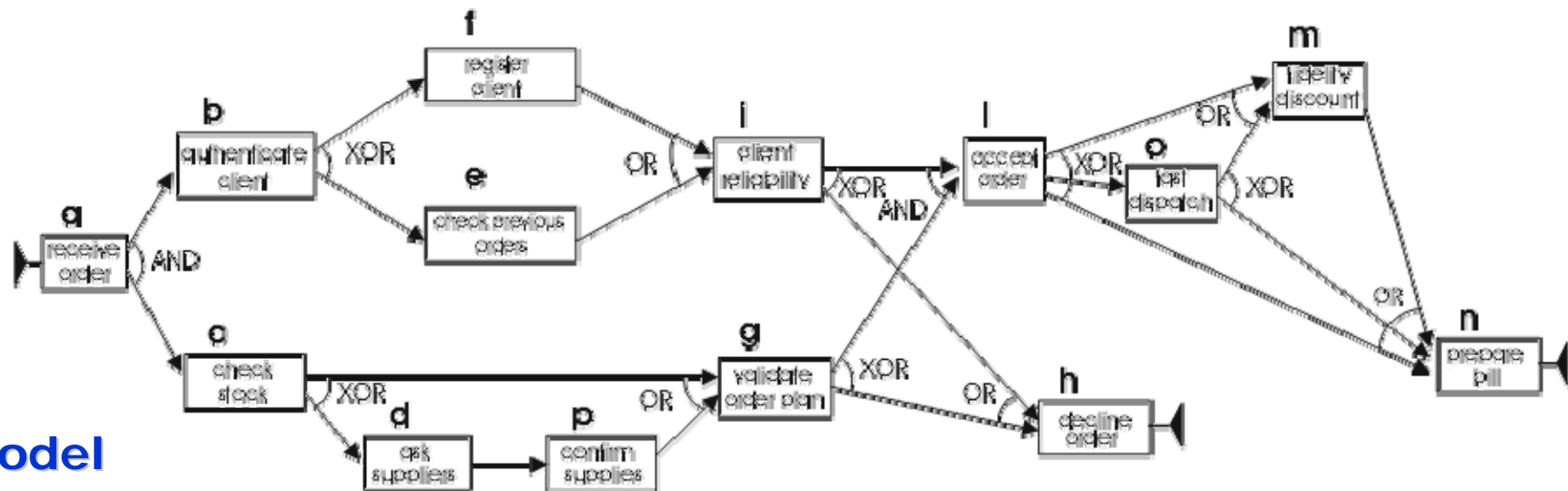
$s_1: acdbfghi$ $s_5: abicglmn$ $s_9: abficgln$ $s_{13}: abcidglmn$
 $s_2: abficdgh$ $s_6: acbiglon$ $s_{10}: acgbfilon$ $s_{14}: acdbiglmn$
 $s_3: acgbfih$ $s_7: acbgilomn$ $s_{11}: abcfdigln$ $s_{15}: abcdgilmn$
 $s_4: abcgiln$ $s_8: abcfgilon$ $s_{12}: acdbfigln$ $s_{16}: acbidgln$

Event Log



Basic induction scheme

1. Mine a **Dependency Graph** encoding a minimal set of precedence links
2. Mine a set of cardinality (local) constraints, based on simple statistics
3. Introduce support thresholds to handle noisy data



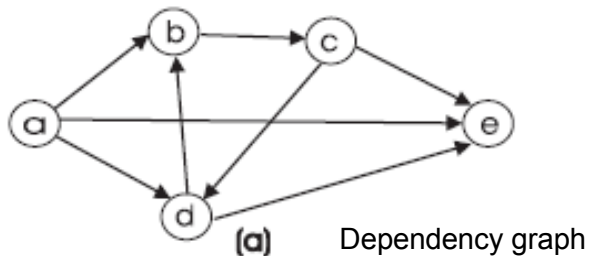
Mined Model

Dependency graph

- Dependency graph for a log L is a graph $D_L = \langle A, E \rangle$ such that

$$E = \{ (a, b) \mid \exists s \in L, i \in \{1, \dots, \text{length}(s)-1\} \text{ s.t. } a = s[i] \wedge b = s[i+1] \};$$
- Parallel activities
 Two activities a and b are parallel in L , if they occur in some cycle of D_L
- Precedence
 The activity a precedes b in L , denoted with $a \rightarrow b$, if a and b are not parallel and there is a path from a to b in D_L

Example: Log $L = \{abcde, adbce, ae\}$



- a, b and c are parallel activities in L ;
- $a \rightarrow b$;
- $b \rightarrow e$;

Basic Workflow Discovery scheme

Input: A log \mathcal{L}_P .

Output: A workflow schema $\mathcal{WS} = \langle A, E, a_0, A_F, Join, Fork \rangle$.

Method: Perform the following steps:

```

1   $\langle A, E \rangle := D_{\mathcal{L}_P}$ ; //nodes and edges are initially those of the dependency graph
2  for each  $(a, b) \in E$  s.t.  $a$  and  $b$  are parallel in  $\mathcal{L}_P$  do //remove cycles
3       $E := E - \{(a, b)\}$ ;
4      for each  $s \in \mathcal{L}_P$  s.t.  $\{a, b\} \subseteq tasks(s)$  do //update edges
5           $pre := s[i]$ , where  $s[i] \rightarrow a \wedge s[i] \rightarrow b$  and not exists  $s[k]$  with  $k > i$  s.t.  $s[k] \rightarrow a \wedge s[k] \rightarrow b$ ;
6           $E := E \cup \{(pre, a)\} \cup \{(pre, b)\}$ ;
7           $post := s[j]$ , where  $a \rightarrow s[j] \wedge b \rightarrow s[j]$  and not exists  $s[h]$  with  $h < j$  s.t.  $a \rightarrow s[h] \wedge b \rightarrow s[h]$ ;
8           $E := E \cup \{(a, post)\} \cup \{(b, post)\}$ ;
9      end for
10 end for
11  $a_0 := s[1]$ , no matter of which trace  $s \in \mathcal{L}_P$  is selected;  $A_F := \{a \in A \mid \nexists b \in A \text{ s.t. } a \rightarrow b\}$ ;
12 for each  $a \in A$  do //construction of local constraints
13     if  $\forall s \in \mathcal{L}_P$  s.t.  $a \in tasks(s)$ , it holds that  $\forall c$  s.t.  $(a, c) \in E, c \in tasks(s)$  then  $Fork(a) = AND$ ;
14     else if  $\forall s \in \mathcal{L}_P$  s.t.  $a \in tasks(s), |\{c \mid (a, c) \in E \wedge c \in tasks(s)\}| = 1$  then  $Fork(a) = XOR$ ;
15     else  $Fork(a) = OR$ ;
16     if  $\forall s \in \mathcal{L}_P$  s.t.  $a \in tasks(s), (c, a) \in E \Rightarrow c \in tasks(s)$  then  $Join(a) = AND$ ;
17     else  $Join(a) = OR$ ;
18 end for
19 return  $\langle A, E, a_0, A_F, Join, Fork \rangle$ ;

```

Build the
dependency graph
And make it
coincide with the
initial CFG model

Removal of
cycles
Remove, from E ,
all edge between
parallel activities

Connect the vertices of
the the edge, with
preceding nodes.

Connect the vertices of
the removed edge with
following nodes

Identification of the first
node a_0 and the set of
final nodes A_F .

Basic Workflow Discovery Scheme

Input: A log \mathcal{L}_P .

Output: A workflow schema $\mathcal{WS} = \langle A, E, a_0, A_F, \text{Join}, \text{Fork} \rangle$.

Method: Perform the following steps:

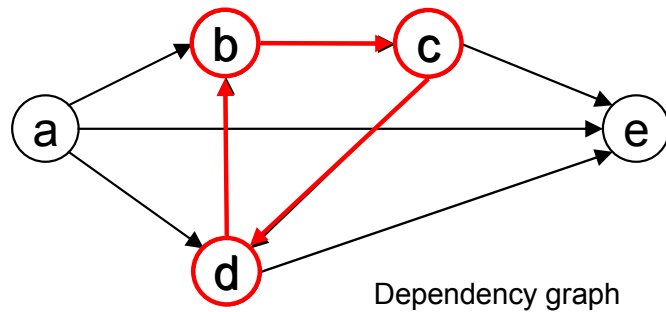
```

1   $\langle A, E \rangle := D_{\mathcal{L}_P}$ ;           //nodes and edges are initially those of the dependency graph
2  for each  $(a, b) \in E$  s.t.  $a$  and  $b$  are parallel in  $\mathcal{L}_P$  do           //remove cycles
3     $E := E - \{(a, b)\}$ ;
4    for each  $s \in \mathcal{L}_P$  s.t.  $\{a, b\} \subseteq \text{tasks}(s)$  do           //update edges
5       $pre := s[i]$ , where  $s[i] \rightarrow a \wedge s[i] \rightarrow b$  and not exists  $s[k]$  with  $k > i$  s.t.  $s[k] \rightarrow a \wedge s[k] \rightarrow b$ ;
6       $E := E \cup \{(pre, a)\} \cup \{(pre, b)\}$ ;
7       $post := s[j]$ , where  $a \rightarrow s[j] \wedge b \rightarrow s[j]$  and not exists  $s[h]$  with  $h < j$  s.t.  $a \rightarrow s[h] \wedge b \rightarrow s[h]$ ;
8       $E := E \cup \{(a, post)\} \cup \{(b, post)\}$ ;
9    end for
10  end for
11   $a_0 := s[1]$ , no matter of which trace  $s \in \mathcal{L}_P$  is selected;    $A_F := \{a \in A \mid \nexists b \in A \text{ s.t. } a \rightarrow b\}$ ;
12  for each  $a \in A$  do           //construction of local constraints
13    if  $\forall s \in \mathcal{L}_P$  s.t.  $a \in \text{tasks}(s)$ , it holds that  $\forall c$  s.t.  $(a, c) \in E, c \in \text{tasks}(s)$  then  $\text{Fork}(a) = \text{AND}$ ;
14    else if  $\forall s \in \mathcal{L}_P$  s.t.  $a \in \text{tasks}(s), |\{c \mid (a, c) \in E \wedge c \in \text{tasks}(s)\}| = 1$  then  $\text{Fork}(a) = \text{XOR}$ ;
15    else  $\text{Fork}(a) = \text{OR}$ ;
16    if  $\forall s \in \mathcal{L}_P$  s.t.  $a \in \text{tasks}(s), (c, a) \in E \Rightarrow c \in \text{tasks}(s)$  then  $\text{Join}(a) = \text{AND}$ ;
17    else  $\text{Join}(a) = \text{OR}$ ;
18  end for
19  return  $\langle A, E, a_0, A_F, \text{Join}, \text{Fork} \rangle$ ;

```

Derive local constraints

Example: Algorithm simulation



Precedences:

a→b	b→c	c→b	d→b
a→c	b→d	c→d	d→c
a→d	b→e	c→e	d→e
a→e			

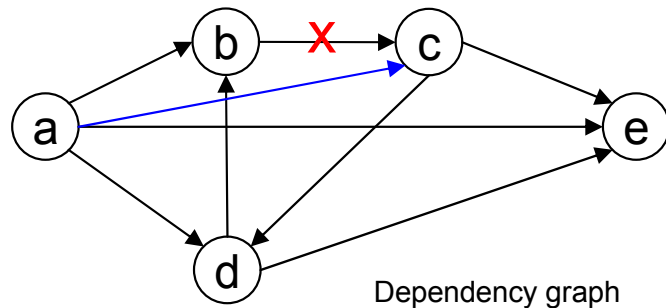
Parallel activities:

- b, c, d

Edges to remove:

- (b, c),
- (b, d),
- (c, d).

Example: Algorithm simulation



Edges to remove: (b, c), (d, b), (c, d).

$\log L = \{\text{abcde}, \text{adbce}, \text{ae}\},$

pre

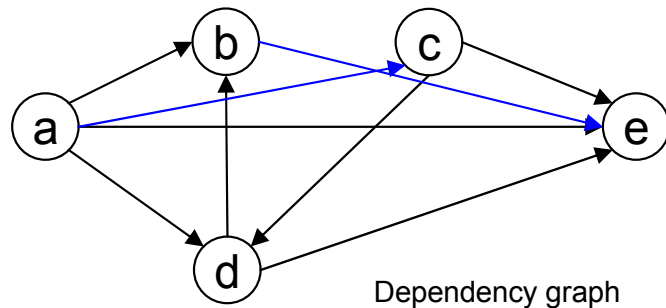
a→b	b→c	c→b	d→b
a→c	b→d	c→d	d→c
a→d	b→e	c→e	d→e
a→e			

$E := \{(a, b), (a, d), (a, e),$
 $(\text{b, c}), (c, d), (c, e), \quad \cup \{(a, c)\};$
 $(d, b), (d, e)\}$

```

2   for each (a, b) ∈ E s.t. a and b are parallel in  $\mathcal{L}_P$  do           //remove cycles
3      $E := E - \{(a, b)\};$ 
4     for each  $s \in \mathcal{L}_P$  s.t.  $\{a, b\} \subseteq \text{tasks}(s)$  do           //update edges
5        $pre := s[i]$ , where  $s[i] \rightarrow a \wedge s[i] \rightarrow b$  and not exists  $s[k]$  with  $k > i$  s.t.  $s[k] \rightarrow a \wedge s[k] \rightarrow b$ ;
6        $E := E \cup \{(pre, a)\} \cup \{(pre, b)\};$ 
7        $post := s[j]$ , where  $a \rightarrow s[j] \wedge b \rightarrow s[j]$  and not exists  $s[h]$  with  $h < j$  s.t.  $a \rightarrow s[h] \wedge b \rightarrow s[h]$ ;
8        $E := E \cup \{(a, post)\} \cup \{(b, post)\};$ 
9     end for
10  end for
  
```

Example: Algorithm simulation



Edges to remove: (b, c), (d, b), (c, d).

$\log L = \{abcde, adbce, ae\}$,

post

a→b	b→c	c→b	d→b
a→c	b→d	c→d	d→c
a→d	b→e	c→e	d→e
a→e			

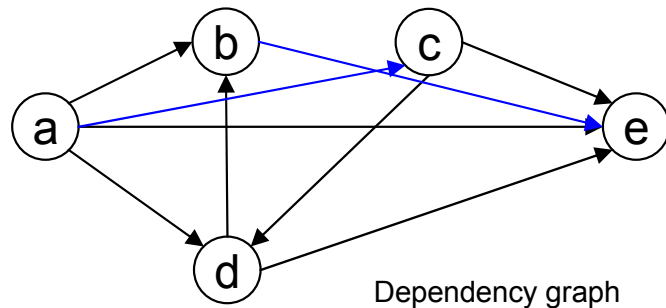
$E := \{(a, b), (a, d), (a, e),$
 $(\cancel{b, c}), (c, d), (c, e), \quad \cup \{(a, c)\} \cup \{(b, e)\}$
 $(d, b), (d, e)\}$

```

4   for each  $s \in \mathcal{L}_P$  s.t.  $\{a, b\} \subseteq \text{tasks}(s)$  do           //update edges
5        $pre := s[i]$ , where  $s[i] \rightarrow a \wedge s[i] \rightarrow b$  and not exists  $s[k]$  with  $k > i$  s.t.  $s[k] \rightarrow a \wedge s[k] \rightarrow b$ ;
6        $E := E \cup \{(pre, a)\} \cup \{(pre, b)\}$ ;
7        $post := s[j]$ , where  $a \rightarrow s[j] \wedge b \rightarrow s[j]$  and not exists  $s[h]$  with  $h < j$  s.t.  $a \rightarrow s[h] \wedge b \rightarrow s[h]$ ;
8        $E := E \cup \{(a, post)\} \cup \{(b, post)\}$ ;
9   end for

```


Example: Algorithm simulation



Edges to remove: (b, c), (d, b), (c, d).

$\log L = \{abcde, adbce, ae\}$,

pre

a→b	b→c	c→b	d→b
a→c	b→d	c→d	d→c
a→d	b→e	c→e	d→e
a→e			

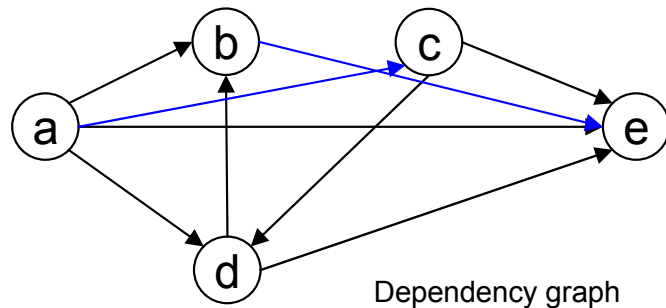
$E := \{(a, b), (a, d), (a, e),$
 $(\cancel{b, c}), (c, d), (c, e), \quad \cup \{(a, c)\} \cup \{(b, e)\}$
 $(d, b), (d, e)\}$

```

4   for each  $s \in \mathcal{L}_P$  s.t.  $\{a, b\} \subseteq \text{tasks}(s)$  do           //update edges
5        $pre := s[i]$ , where  $s[i] \rightarrow a \wedge s[i] \rightarrow b$  and not exists  $s[k]$  with  $k > i$  s.t.  $s[k] \rightarrow a \wedge s[k] \rightarrow b$ ;
6        $E := E \cup \{(pre, a)\} \cup \{(pre, b)\}$ ;
7        $post := s[j]$ , where  $a \rightarrow s[j] \wedge b \rightarrow s[j]$  and not exists  $s[h]$  with  $h < j$  s.t.  $a \rightarrow s[h] \wedge b \rightarrow s[h]$ ;
8        $E := E \cup \{(a, post)\} \cup \{(b, post)\}$ ;
9   end for

```

Example: Algorithm simulation



Edges to remove: (b, c), (d, b), (c, d).

$\log L = \{abcde, adbce, ae\}$,

post

a→b	b→c	c→b	d→b
a→c	b→d	c→d	d→c
a→d	b→e	c→e	d→e
a→e			

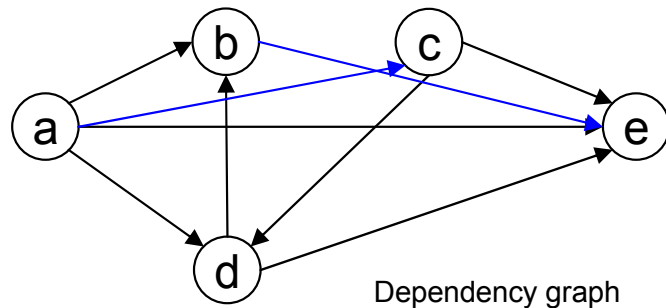
$E := \{(a, b), (a, d), (a, e),$
 $(\cancel{b, c}), (c, d), (c, e), \quad \cup \{(a, c)\} \cup \{(b, e)\}$
 $(d, b), (d, e)\}$

```

4   for each  $s \in \mathcal{L}_P$  s.t.  $\{a, b\} \subseteq \text{tasks}(s)$  do           //update edges
5        $pre := s[i]$ , where  $s[i] \rightarrow a \wedge s[i] \rightarrow b$  and not exists  $s[k]$  with  $k > i$  s.t.  $s[k] \rightarrow a \wedge s[k] \rightarrow b$ ;
6        $E := E \cup \{(pre, a)\} \cup \{(pre, b)\}$ ;
7        $post := s[j]$ , where  $a \rightarrow s[j] \wedge b \rightarrow s[j]$  and not exists  $s[h]$  with  $h < j$  s.t.  $a \rightarrow s[h] \wedge b \rightarrow s[h]$ ;
8        $E := E \cup \{(a, post)\} \cup \{(b, post)\}$ ;
9   end for

```

Example: Algorithm simulation



Edges to remove: (b, c), (d, b), (c, d).

$\log L = \{abcde, adbce, ae\}$

a→b	b→c	c→b	d→b
a→c	b→d	c→d	d→c
a→d	b→e	c→e	d→e
a→e			

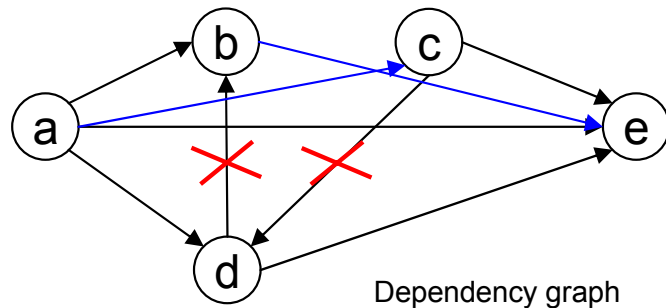
$E := \{(a, b), (a, d), (a, e),$
 $(\cancel{b, c}), (c, d), (c, e), \quad \cup \{(a, c)\} \cup \{(b, e)\}$
 $(d, b), (d, e)\}$

```

4   for each  $s \in \mathcal{L}_P$  s.t.  $\{a, b\} \subseteq \text{tasks}(s)$  do           //update edges
5        $pre := s[i]$ , where  $s[i] \rightarrow a \wedge s[i] \rightarrow b$  and not exists  $s[k]$  with  $k > i$  s.t.  $s[k] \rightarrow a \wedge s[k] \rightarrow b$ ;
6        $E := E \cup \{(pre, a)\} \cup \{(pre, b)\}$ ;
7        $post := s[j]$ , where  $a \rightarrow s[j] \wedge b \rightarrow s[j]$  and not exists  $s[h]$  with  $h < j$  s.t.  $a \rightarrow s[h] \wedge b \rightarrow s[h]$ ;
8        $E := E \cup \{(a, post)\} \cup \{(b, post)\}$ ;
9   end for

```

Example: Algorithm simulation



Edges to remove: (b, c), (d, b), (c, d).

$\log L = \{abcde, adbce, ae\},$

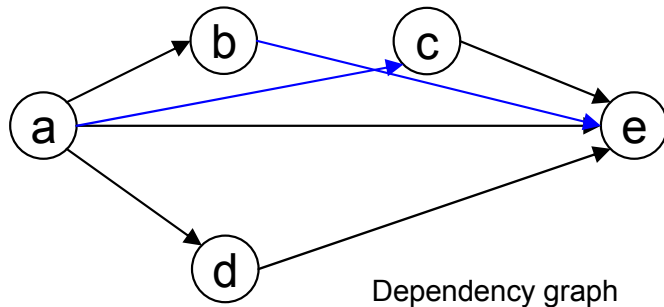
a→b	b→c	c→b	d→b
a→c	b→d	c→d	d→c
a→d	b→e	c→e	d→e
a→e			

$E := \{(a, b), (a, d), (a, e),$
 $(\cancel{b, c}), (c, d), (c, e), \quad \cup \{(a, c)\} \cup \{(b, e)\}$
 $(\cancel{d, b}), (\cancel{d, e})\}$

```

2   for each  $(a, b) \in E$  s.t.  $a$  and  $b$  are parallel in  $\mathcal{L}_P$  do           //remove cycles
3        $E := E - \{(a, b)\};$ 
4       for each  $s \in \mathcal{L}_P$  s.t.  $\{a, b\} \subseteq \text{tasks}(s)$  do           //update edges
5            $pre := s[i]$ , where  $s[i] \rightarrow a \wedge s[i] \rightarrow b$  and not exists  $s[k]$  with  $k > i$  s.t.  $s[k] \rightarrow a \wedge s[k] \rightarrow b$ ;
6            $E := E \cup \{(pre, a)\} \cup \{(pre, b)\};$ 
7            $post := s[j]$ , where  $a \rightarrow s[j] \wedge b \rightarrow s[j]$  and not exists  $s[h]$  with  $h < j$  s.t.  $a \rightarrow s[h] \wedge b \rightarrow s[h]$ ;
8            $E := E \cup \{(a, post)\} \cup \{(b, post)\};$ 
9       end for
10  end for
  
```

Example: Algorithm simulation



Edges to remove: (b, c), (d, b), (c, d).

$\log L = \{abcde, adbce, ae\},$

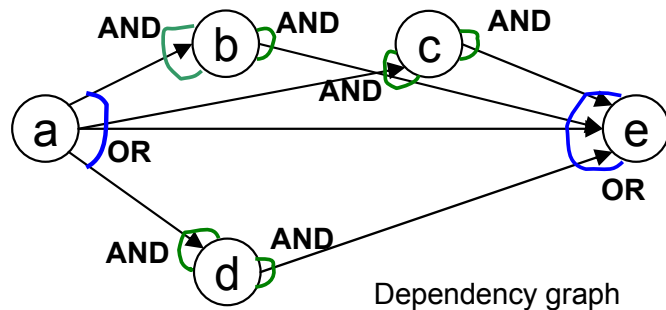
a→b	b→c	c→b	d→b
a→c	b→d	c→d	d→c
a→d	b→e	c→e	d→e
a→e			

$E := \{(a, b), (a, d), (a, e),$
 $(\cancel{b, c}), (c, d), (c, e), \quad \cup \{(a, c)\} \quad \cup \{(b, e)\}$
 $(\cancel{d, b}), (\cancel{d, e})\}$

- $a_0 := a;$
- $A_F := \{e\}$

11 $a_0 := s[1]$, no matter of which trace $s \in \mathcal{L}_P$ is selected; $A_F := \{a \in A \mid \nexists b \in A \text{ s.t. } a \rightarrow b\};$

Example: Algorithm simulation



$\log L = \{abcde, adbce, ae\},$

$A = \{a, b, c, d, e\},$



$E := \{(a, b), (a, d), (a, e),$
 ~~(b, c)~~ , $(c, d), (c, e),$ $\cup \{(a, c)\}$ $\cup \{(b, e)\}$
 ~~(d, b)~~ , ~~$(d, e)\}$~~

- $a_0 := a;$
- $A_F := \{e\}$

```

12  for each  $a \in A$  do           //construction of local constraints
13  → if  $\forall s \in \mathcal{L}_P$  s.t.  $a \in tasks(s)$ , it holds that  $\forall c$  s.t.  $(a, c) \in E, c \in tasks(s)$  then  $Fork(a) = AND;$ 
14     else if  $\forall s \in \mathcal{L}_P$  s.t.  $a \in tasks(s), |\{c \mid (a, c) \in E \wedge c \in tasks(s)\}| = 1$  then  $Fork(a) = XOR;$ 
15  → else  $Fork(a) = OR;$ 
16  → if  $\forall s \in \mathcal{L}_P$  s.t.  $a \in tasks(s), (c, a) \in E \Rightarrow c \in tasks(s)$  then  $Join(a) = AND;$ 
17     else  $Join(a) = OR;$ 
18  end for

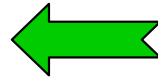
```

Outline

- Part I – Introduction to Process Mining
 - Context, motivation and goal
 - General characteristics of the analyzed processes and logs
 - Classification of Process Mining approaches
- Part II – Workflow discovery
 - Basic CFG induction algorithm
 - Other algorithms (α -algorithm, Heuristic Miner, Fuzzy mining)
- Part III – Beyond the control-flow mining perspective
 - Organizational mining
 - Social net mining
 - Extension algorithms
- Part IV – Evaluation and validation of discovered models
 - Conformance Check
 - Log-based property verification
- Part V – Clustering-based Process Mining
 - Discovery of hierarchical process models
 - Discovery of process taxonomies
 - Outlier detection

Workflow discovery algorithms

- Multi-phase PM
- α -algorithm
- Heuristics Miner
- Genetic PM
- Fuzzy Miner



Outline



- Part I – Introduction to Process Mining
 - Context, motivation and goal
 - General characteristics of the analyzed processes and logs
 - Classification of Process Mining approaches
- Part II – Workflow discovery
 - Basic CFG induction algorithm
 - Other algorithms (α -algorithm, Heuristic Miner, Fuzzy mining)
- Part III – Beyond the control-flow perspective
 - Organizational mining
 - Social net mining
 - Extension techniques
- Part IV – Evaluation and validation of discovered models
 - Conformance Check
 - Log-based property verification
- Part V – Advanced Process Mining approaches
 - Discovery of hierarchical process models
 - Discovery of process taxonomies
 - Outlier detection in a process mining setting

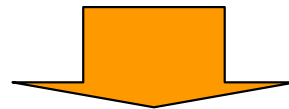
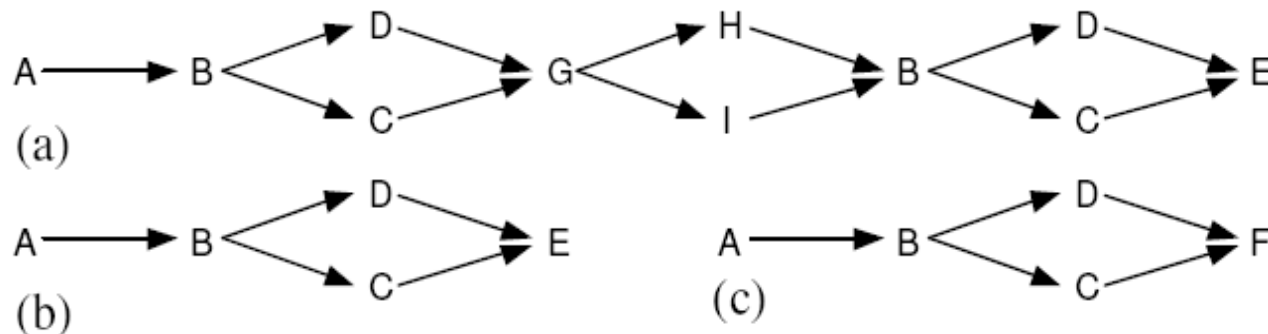
Multi-phase mining

■ Main steps

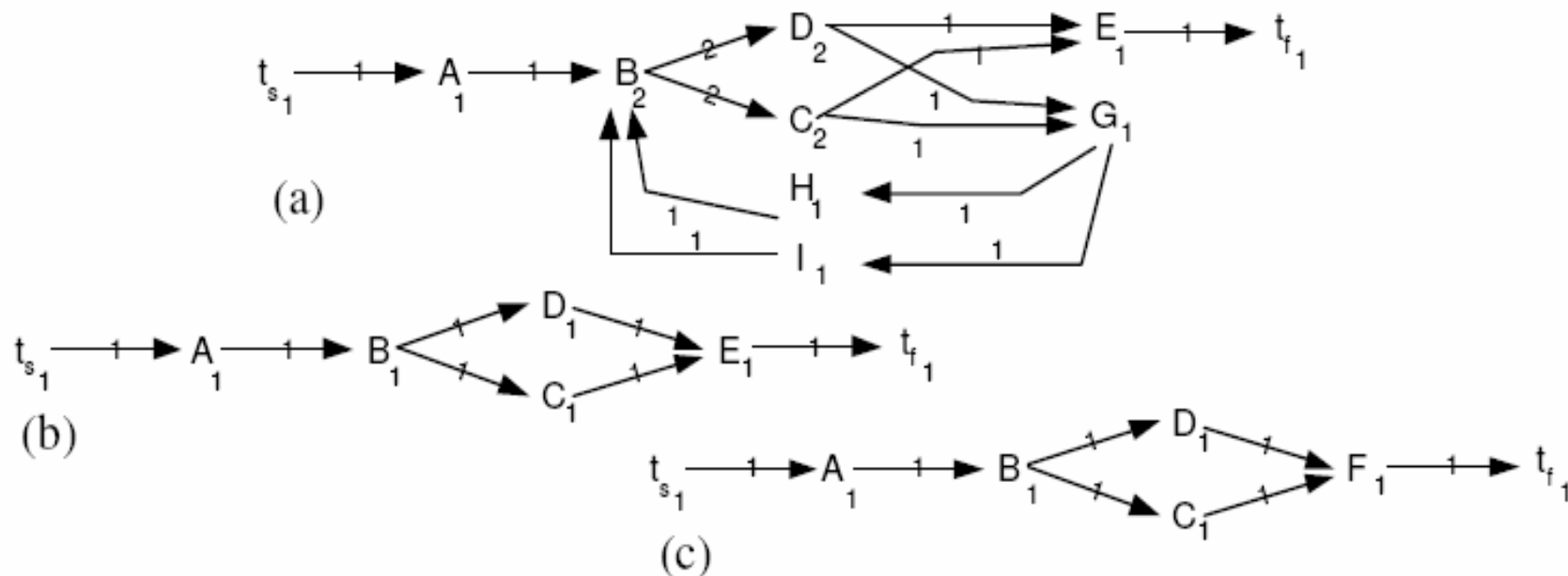
- Convert each log trace into an *execution graph*, where each node corresponds to the execution of a task
 - a task label can appear multiple times!
- Convert each instance graph into an *instance graph*
 - each node is associated with a single task
 - both nodes and edges are labelled with occurrence counters
 - a fictive start node and a fictive final node are introduced
- Merge the *instance graphs* into an *aggregated graph model*
 - The model is simply the union of all the *instance graphs*
 - Arc/node counters are summed up
- Convert the CFG model into an EPC

Multi-phase mining: Example

- Execution graphs (acyclic):



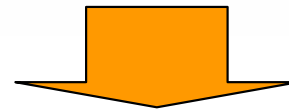
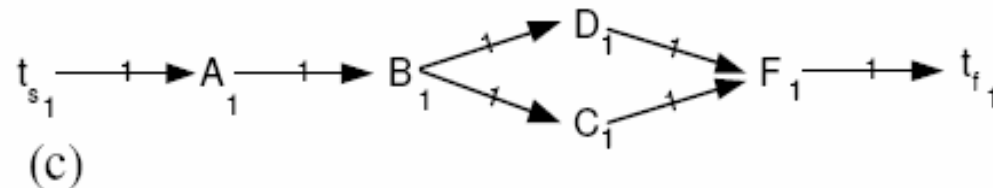
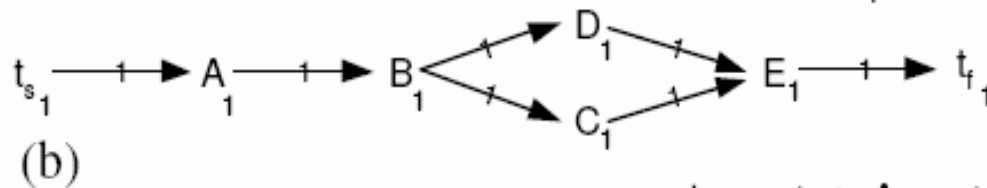
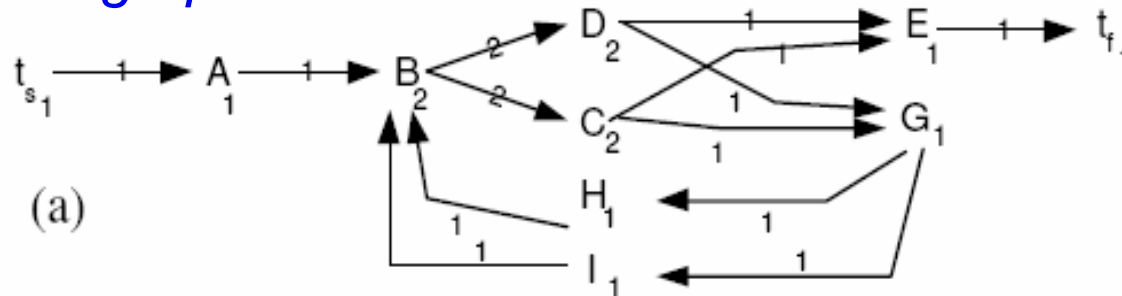
- Instance graphs (some cycles can be created)



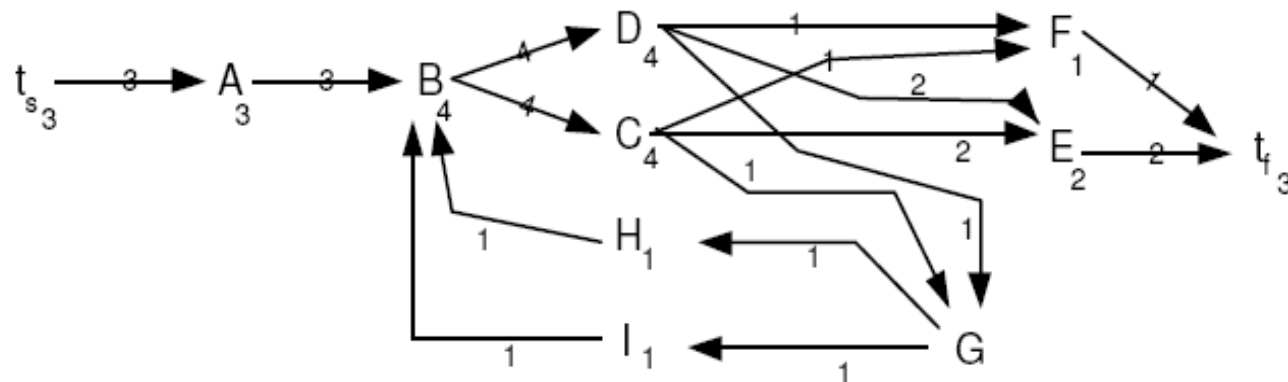
any log
node)

Multi-phase mining: Example (2)

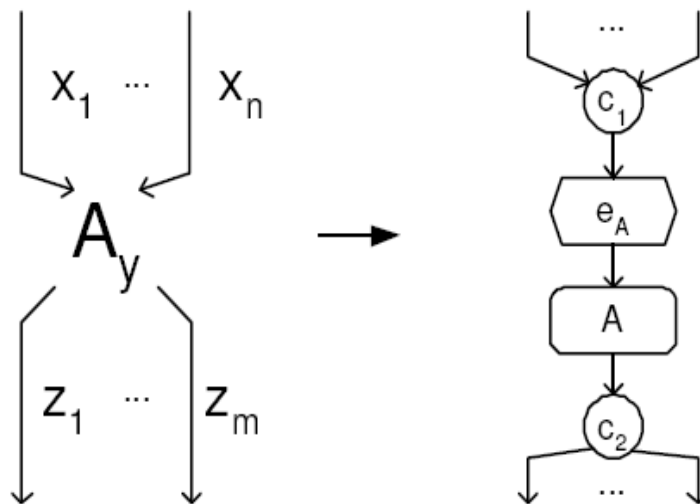
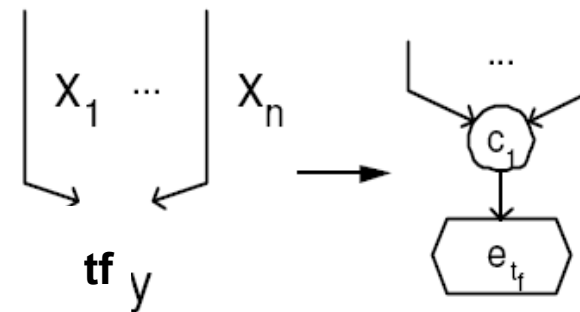
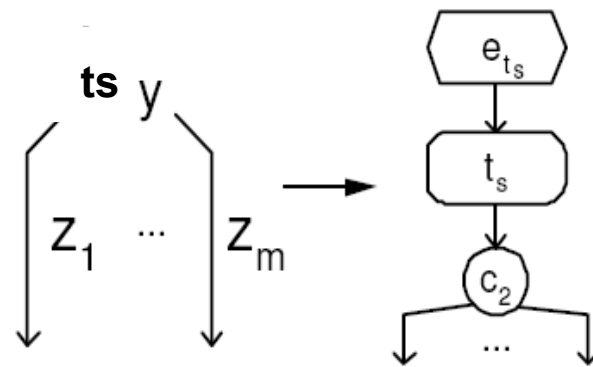
■ Instance graphs:



■ Aggregated graph model:



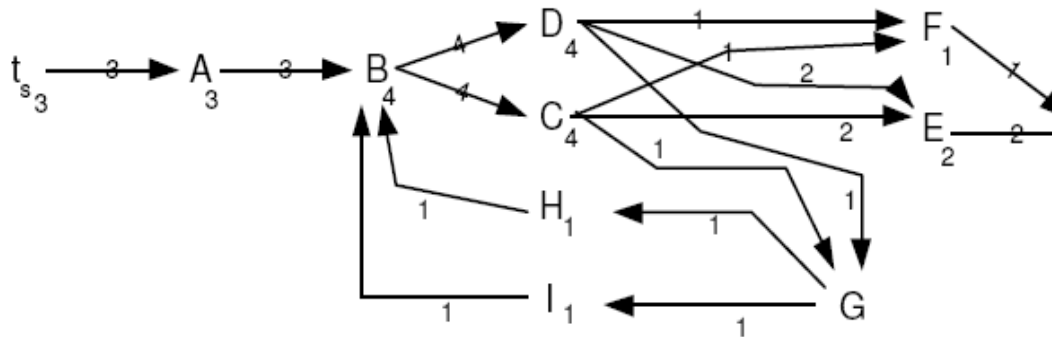
Multi-phase mining: deriving an EPC



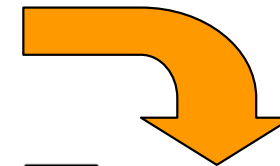
- If $\sum_{i=1}^n (x_i) = y$ then $c_1 = XOR$,
- If $\forall_{i=1}^n (x_i) = y$ then $c_1 = AND$,
- Else $c_1 = OR$.

- If $\sum_{i=1}^m (z_i) = y$ then $c_2 = XOR$,
- If $\forall_{i=1}^m (z_i) = y$ then $c_2 = AND$,
- Else $c_2 = OR$.

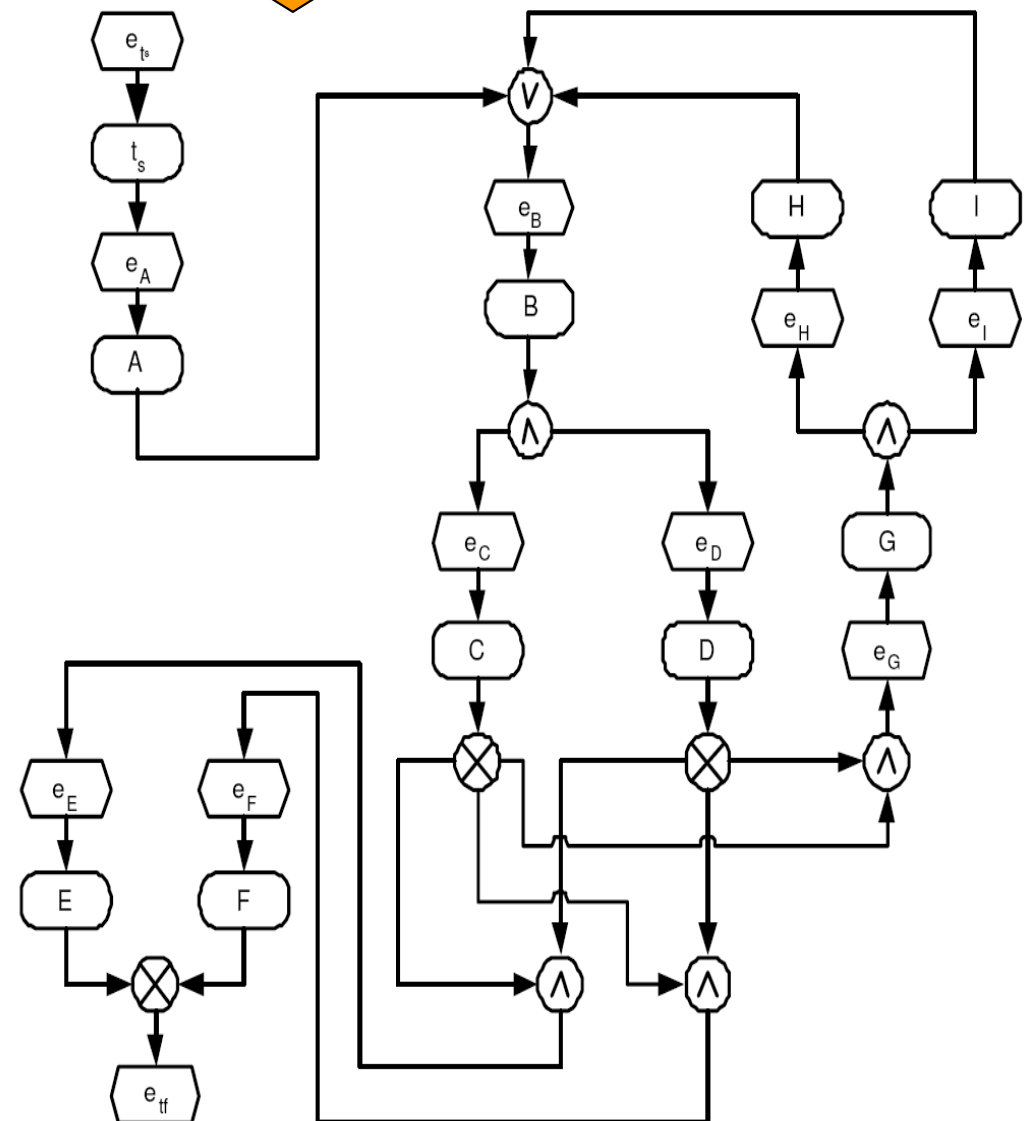
Multi-phase mining: Example (3)



Aggregated graph model

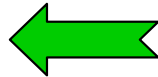


EPC model



Workflow discovery algorithms

- Multi-phase PM
- α -algorithm
- Heuristics Miner
- Genetic PM
- Fuzzy Miner



Outline



- Part I – Introduction to Process Mining
 - Context, motivation and goal
 - General characteristics of the analyzed processes and logs
 - Classification of Process Mining approaches
- Part II – Workflow discovery
 - Basic CFG induction algorithm
 - Other algorithms (α -algorithm, Heuristic Miner, Fuzzy mining)
- Part III – Beyond the control-flow perspective
 - Organizational mining
 - Social net mining
 - Extension techniques
- Part IV – Evaluation and validation of discovered models
 - Conformance Check
 - Log-based property verification
- Part V – Advanced Process Mining approaches
 - Discovery of hierarchical process models
 - Discovery of process taxonomies
 - Outlier detection in a process mining setting

α -algorithm

Output: a Petri net

Method:

- Read the input log
- Get the set of tasks
- Infer a set of ordering relations
- Build the net based on inferred relations
- Return the net

α -algorithm - Ordering Relations $>, \rightarrow, ||, \#$

- **Direct succession:**

$x > y$ iff for some case x is directly followed by y

- **Causality:**

$x \rightarrow y$ iff $x > y$ and not $y > x$

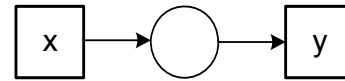
- **Parallel:**

$x || y$ iff $x > y$ and $y > x$

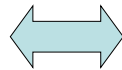
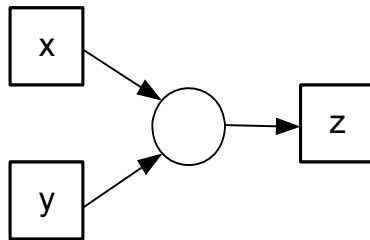
- **Unrelated:**

$x \# y$ iff not $x > y$ and not $y > x$

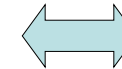
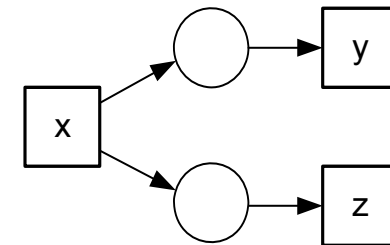
From the ordering relations to the Petri net



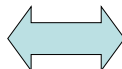
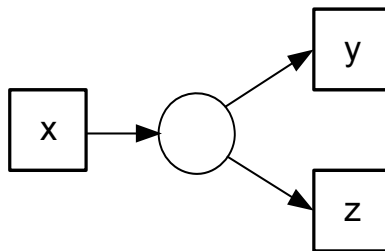
$x \rightarrow y$



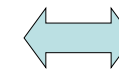
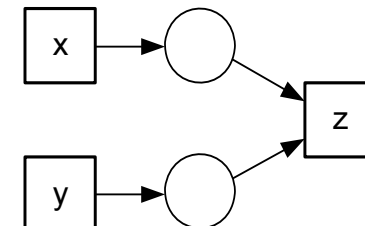
$x \rightarrow z, y \rightarrow z, \text{ and } x \# y$



$x \rightarrow y, x \rightarrow z, \text{ and } y \parallel z$



$x \rightarrow y, x \rightarrow z, \text{ and } y \# z$



$x \rightarrow z, y \rightarrow z, \text{ and } x \parallel y$

α -algorithm - Formalization

Let W be a workflow log over T . $\alpha(W)$ is defined as follows.

1. $T_W = \{ t \in T \mid \exists_{\sigma \in W} t \in \sigma \},$
2. $T_I = \{ t \in T \mid \exists_{\sigma \in W} t = first(\sigma) \},$
3. $T_O = \{ t \in T \mid \exists_{\sigma \in W} t = last(\sigma) \},$
4. $X_W = \{ (A,B) \mid A \subseteq T_W \wedge B \subseteq T_W \wedge \forall_{a \in A} \forall_{b \in B} a \rightarrow_W b \wedge \forall_{a_1, a_2 \in A} a_1 \#_W a_2 \wedge \forall_{b_1, b_2 \in B} b_1 \#_W b_2 \},$
5. $Y_W = \{ (A,B) \in X \mid \forall_{(A',B') \in X} A \subseteq A' \wedge B \subseteq B' \Rightarrow (A,B) = (A',B') \},$
6. $P_W = \{ p_{(A,B)} \mid (A,B) \in Y_W \} \cup \{ i_W, o_W \},$
7. $F_W = \{ (a, p_{(A,B)}) \mid (A,B) \in Y_W \wedge a \in A \} \cup \{ (p_{(A,B)}, b) \mid (A,B) \in Y_W \wedge b \in B \} \cup \{ (i_W, t) \mid t \in T_I \} \cup \{ (t, o_W) \mid t \in T_O \},$ and
8. $\alpha(W) = (P_W, T_W, F_W).$