Using Maximum Entropy for Text Classification

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Abstract

This paper proposes the use of maximum entropy techniques for text classification. Maximum entropy is a probability distribution estimation technique widely used for a variety of natural language tasks, such as language modeling, part-of-speech tagging, and text segmentation. The underlying principle of maximum entropy is that without external knowledge, one should prefer distributions that are uniform. Constraints on the distribution, derived from labeled training data, inform the technique where to be minimally non-uniform. The maximum entropy formulation has a unique solution which can be found by the improved iterative scaling algorithm. In this paper, maximum entropy is used for text classification by estimating the conditional distribution of the class variable given the document. In experiments on several text datasets we compare accuracy to naive Bayes and show that maximum entropy is sometimes significantly better, but also sometimes worse. Much future work remains, but the results indicate that maximum entropy is a promising technique for text classification.

1 Introduction

A variety of techniques for supervised learning algorithms have demonstrated reasonable performance for text classification; a non-exhaustive list includes naive Bayes [Lewis, 1998; McCallum and Nigam, 1998; Sahami, 1996], k-nearest neighbor [Yang, 1999], support vector machines [Joachims, 1998; Dumais et al., 1998], boosting [Schapire and Singer, 1999] and rule learning algorithms [Cohen and Singer, 1996; Slattery and Craven, 1998]. Among these, however, no single technique has proven to consistently outperform the others across many domains.

This paper explores the use of maximum entropy for text classification as an alternative to previously used text classification algorithms. Maximum entropy has already been widely used for a variety of natural language tasks, including language modeling [Chen and Rosenfeld, 1999; Rosenfeld, 1994], text segmentation [Beeferman et al., 1999], part-of-speech tagging [Ratnaparkhi, 1996], and prepositional phrase attachment [Ratnaparkhi et al., 1994]. Maximum entropy has been shown to be a viable and competitive algorithm in these domains.

Maximum entropy is a general technique for estimating probability distributions from data. The over-riding principle in maximum entropy is that when nothing is known, the distribution should be as uniform as possible, that is, have maximal entropy. Labeled training data is used to derive a set of constraints for the model that characterize the class-specific expectations for the distribution. Constraints are represented as expected values of “features,” any real-valued function of an example. The improved iterative scaling algorithm finds the maximum entropy distribution that is consistent with the given constraints.

In our text classification scenario, maximum entropy estimates the conditional distribution of the class label given a document. A document is represented by a set of word count features. The labeled training data is used to estimate the expected value of these word counts on a class-by-class basis. Improved iterative scaling finds a text classifier of an exponential form that is consistent with the constraints from the labeled data.

Our experimental results show that maximum entropy is a technique that warrants further investigation for text classification. On one data set, for example, maximum entropy reduces classification error by more than 40% compared to naive Bayes. On other data sets, basic maximum entropy does not perform as well as naive Bayes. Here, there is evidence that basic maximum entropy suffers from overfitting and poor feature selection. When a prior is applied to maximum entropy, performance is improved in these cases. Overall, maximum entropy performs better than naive Bayes on two of three data sets. Many areas for further investigation exist which may improve performance even further. These include more appropriate feature selection, using bigrams and phrases as features, and adjusting the prior based on the sparsity of the data.

This paper proceeds as follows. Section 2 presents the general framework for maximum entropy for estimating
conditional distributions. Then, the specific application of maximum entropy to text classification is discussed in Section 3. Related work is presented in Section 4. Experimental results are presented in Section 5. Finally, Section 6 discusses plans for future work.

2 Maximum Entropy

The motivating idea behind maximum entropy is that one should prefer the most uniform models that also satisfy any given constraints. For example, consider a four-way text classification task where we are told only that on average 40% of documents with the word “professor” in them are in the faculty class. Intuitively, when given a document with “professor” in it, we would say it has a 40% chance of being a faculty document, and a 20% chance for each of the other three classes. If a document does not have “professor” we would guess the uniform class distribution, 25% each. This model is exactly the maximum entropy model that conforms to our known constraint. Calculating the model is easy in this example, but when there are many constraints to satisfy, rigorous techniques are needed to find the optimal solution. Csiszár [1996] provides a good tutorial introduction to maximum entropy techniques.

In its most general formulation, maximum entropy can be used to estimate any probability distribution. In this paper we are interested in classification; thus we limit our further discussion to learning conditional distributions from labeled training data. Specifically, we learn the conditional distribution of the class label given a document.

2.1 Constraints and Features

In maximum entropy we use the training data to set constraints on the conditional distribution. Each constraint expresses a characteristic of the training data that should also be present in the learned distribution. We let any real-valued function of the document and the class be a feature, \( f_i(d, c) \). Maximum entropy allows us to restrict the model distribution to have the same expected value for this feature as seen in the training data, \( D \). Thus, we stipulate that the learned conditional distribution \( P(c|d) \) must have the property:

\[
\frac{1}{|D|} \sum_{d \in D} f_i(d, c(d)) = \sum_d P(d) \sum_c P(c|d) f_i(d, c). \tag{1}
\]

In practice, the document distribution \( P(d) \) is unknown, and we are not interested in modeling it. Thus, we use our training data, without class labels, as an approximation to the document distribution, and enforce the constraint:

\[
\frac{1}{|D|} \sum_{d \in D} f_i(d, c(d)) = \frac{1}{|D|} \sum_{d \in D} \sum_c P(c|d) f_i(d, c). \tag{2}
\]

Thus, when using maximum entropy, the first step is to identify a set of feature functions that will be useful for classification. Then, for each feature, measure its expected value over the training data and take this to be a constraint for the model distribution.

2.2 Parametric Form

When constraints are estimated in this fashion, it is guaranteed that a unique distribution exists that has maximum entropy. Moreover, it can be shown [Della Pietra et al., 1997] that the distribution is always of the exponential form:

\[
P(c|d) = \frac{1}{Z(d)} \exp \left( \sum_i \lambda_i f_i(d, c) \right), \tag{3}
\]

where each \( f_i(d, c) \) is a feature, \( \lambda_i \) is a parameter to be estimated and \( Z(d) \) is simply the normalizing factor to ensure a proper probability:

\[
Z(d) = \sum_c \exp \left( \sum_i \lambda_i f_i(d, c) \right). \tag{4}
\]

When the constraints are estimated from labeled training data, the solution to the maximum entropy problem is also the solution to a dual maximum likelihood problem for models of the same exponential form. Additionally, it is guaranteed that the likelihood surface is convex, having a single global maximum and no local maxima. This suggests one possible approach for finding the maximum entropy solution: guess any initial exponential distribution of the correct form as a starting point; then, perform hillclimbing in likelihood space. Since there are no local maxima, this will converge to the maximum likelihood solution for exponential models, which will also be the global maximum entropy solution.

2.3 Improved Iterative Scaling

In this section, we briefly outline the derivation of improved iterative scaling (IIS), a hillclimbing algorithm for calculating the parameters of a maximum entropy classifier given a set of constraints. We also describe the algorithmic details of this procedure. A complete description and derivation of improved iterative is presented by Della Pietra et al. [1997]. This presentation follows that of Berger [1998].

IIS performs hillclimbing in parameter log log likelihood space. Given a set of i.i.d. training data \( D \), we can calculate the log likelihood of an exponential model, \( \Lambda \), using Equation 3:

\[
l(\Lambda|D) = \log \prod_{d \in D} P_\Lambda(c(d)|d) \\
= \sum_{d \in D} \sum_i \lambda_i f_i(d, c(d)) - \sum_{d \in D} \log \sum_c \exp \left( \sum_i \lambda_i f_i(d, c) \right). \tag{5}
\]

At each step IIS must find an incrementally more likely set of parameters. Since the likelihood function is convex, if we can guarantee that IIS succeeds in improving
the likelihood, then we know it will converge to the globally optimal set of parameters—those that are both the maximum likelihood solution for the parametric form, and the solution with the maximal entropy.

We start from any initial vector of parameters \( \Lambda \); at each step we improve \( \Lambda \), by setting it equal to \( \Lambda + \Delta \), which will have a higher likelihood. Thus, at each step, we want to find a \( \Delta \) such that the difference in likelihoods is positive:

\[
l(\Lambda + \Delta |D) - l(\Lambda |D) > 0. \tag{6}
\]

Using the inequality \(- \log(x) \geq 1 - x\) and Jensen’s inequality, we can bound this expression from below with an auxiliary function we call \( B \):

\[
l(\Lambda + \Delta |D) - l(\Lambda |D) \geq B = \sum_{d \in D} \delta_i f_i(d, c(d)) - \sum_c P_\Lambda(c|d) \exp(f_\Lambda(d, c) \delta_i \sum_i f_i(d, c)) \tag{7}
\]

where \( f_\Lambda(d, c) \) is the sum of all features in training instance \( d \):

\[
f_\Lambda(d, c) = \sum_i f_i(d, c). \tag{8}
\]

We can guarantee an increase in likelihood if we can find \( \Delta \) such that \( B \) is positive. We can find the best \( \Delta \) by differentiating \( B \) with respect to the change in each parameter \( \delta_i \) in turn and solving for the maxima:

\[
\frac{\partial B}{\partial \delta_i} = \sum_{d \in D} \left( f_i(d, c(d)) - \sum_c \sum_i \delta_i f_i(d, c) \exp(\delta_i f_\Lambda(d, c)) \right). \tag{9}
\]

Note that it is straightforward to set this equal to zero and solve for the increment \( \delta_i \) for each parameter \( \lambda_i \). In the case where \( f_\Lambda(d, c) \) is constant for all \( d \) and \( c \) (as in experiments in this paper), this can be solved in closed-form. Otherwise, this can be solved with a numeric root-finding procedure, such as Newton’s method. In this case, the polynomial is guaranteed to have only one positive root.

This analysis shows that we can find at each hillclimbing step changes to each \( \lambda_i \) that improve the model likelihood. Since the likelihood is convex, this hillclimbing is guaranteed to converge to the global maximum.

This is the foundation for the improved iterative scaling algorithm, outlined in Table 1. At each step of the iteration we need to estimate class labels \( P_\Lambda(c|d) \) of all documents with the current model. Then, using the class labels we calculate improved model parameters and iterate.

- **Inputs:** A collection \( D \) of labeled documents and a set of feature functions \( f_i \).
- **Set the constraints (Equation 2).** For every feature \( f_i \), estimate its expected value on the training documents.
- **Initialize all the \( \lambda_i \)’s to be zero.**
- **Iterate until convergence:**
  - Calculate the expected class labels for each document with the current parameters, \( P_\Lambda(c|d) \) (Equation 3).
  - For each parameter \( \lambda_i \):
    - Set \( \frac{\partial B}{\partial \delta_i} = 0 \) and solve for \( \delta_i \) (Equation 9).
    - Set \( \lambda_i = \lambda_i + \delta_i \)
- **Output:** A text classifier that takes an unlabeled document and predicts a class label.

<table>
<thead>
<tr>
<th>Table 1: An outline of the Improved Iterative Scaling algorithm for estimating the parameters for maximum entropy.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2.4 Gaussian Prior</strong></td>
</tr>
<tr>
<td>Maximum entropy can suffer from overfitting. The constraints are estimated from labeled training data, and, like other learning algorithms, when data is sparse, overfitting can occur. With too little data, the expected value of a feature in the training data may be far from the true value. By introducing a prior on the model, overfitting can be reduced and performance improved.</td>
</tr>
</tbody>
</table>
| To integrate a prior into maximum entropy, we use maximum a posteriori estimation for the exponential model, instead of maximum likelihood estimation. We use a Gaussian prior for the model, with the mean at zero, and a diagonal covariance matrix. This prior favors feature weightings that are closer to zero, that is, are less extreme. The prior probability of the model is just the product over the Gaussian of each feature value \( \lambda_i \) with variance \( \sigma_i^2 \):

\[
\text{P}(\Lambda) = \prod_i \frac{1}{\sqrt{2\pi\sigma_i}} \exp\left(-\frac{\lambda_i^2}{2\sigma_i^2}\right). \tag{10}
\]

Integrating this prior into improved iterative scaling requires adding a single term to the derivative of \( B \) (Equation 9):

\[
\frac{\partial B}{\partial \delta_i} = \lambda_i + \delta_i - \sum_{d \in D} \left( f_i(d, c(d)) - \sum_c P_\Lambda(c|d) f_i(d, c) \exp(\delta_i f_\Lambda(d, c)) \right). \tag{11}
\]

Again, this new formula is easily solved for a maximum with a numeric root-finding procedure, like Newton’s method. Chen and Rosenfeld [1999] have shown that introducing a Gaussian prior on each \( \lambda_i \) improves
performance for language modeling tasks when sparse
data causes overfitting. This paper also derives the up-
date rule given by Equation 11.

3 Maximum Entropy for Text Classification

In order to apply maximum entropy to a domain, we
need to select a set of features to use for setting the con-
straints. For text classification with maximum entropy,
we use word counts as our features. More specifically, in
this paper for each word-class combination we instanti-
ate a feature as:

\[
f_{w,c'}(d, c) = \begin{cases} 
0 & \text{if } c \neq c' \\
\frac{N(d, w)}{N(d)} & \text{Otherwise,} 
\end{cases}
\]  

(12)

where \(N(d, w)\) is the number of times word \(w\) occurs in
document \(d\), and \(N(d)\) is the number of words in \(d\).

With this representation, if a word occurs often in one
class, we would expect the weight for that word-class
pair to be higher than for the word paired with other
classes. In most natural language tasks using maximum
entropy the features are naturally binary features. In
text classification, we expect that features accounting for
the number of times a word occurs should improve clas-
sification. For example, naive Bayes implementations
that use counts outperform implementations that do not
[McCallum and Nigam, 1998]. Note that we use scaled
counts as features instead of simple counts. We initially
choose this representation for computational efficiency,
as it lets us perform each IIS iteration in closed form.
Some implications of this choice are discussed in Sec-
tion 6.

One especially pleasing aspect of maximum entropy is
that it does not suffer from any independence assump-
tions. For example, consider the phrase “Buenos Aires,”
where the two words almost always co-occur, and only
rarely occur by themselves. Naive Bayes will double-
count the evidence of this phrase. Maximum entropy,
on the other hand, will discount the \(\lambda_i\) for each of these
features such that their weight towards classification is
appropriately reduced by half. This is because the con-
straints work over expectations of the counts. One im-
pli cation of this freedom from independence assump-
tions is that bigrams and phrases can be easily added as fea-
tures by maximum entropy, without worry that the fea-
tures are overlapping. Experiments with such expanded
features is a promising area of future work.

4 Related Work

Two other studies have been performed using maximum
entropy for text classification. The first, a study by Rat-
naparkhi [1998], is a very preliminary experiment. In
a comparison between maximum entropy and decision
trees, maximum entropy performs better at classifying
the acq class in the Reuters-21578 data set. Here, bi-
nary features are used instead of counts. We generally
expect that representing counts instead of binary fea-
tures should enhance performance.

A recent study on feature selection and model build-
ing for maximum entropy [Mikheev, 1999] examined text
classification performance on the RAPRA corpus of tech-
nical abstracts. Here, maximum entropy compares fa-
vorably to a smoothed logistic term-weighting model.
Again, features are only binary valued. Interestingly,
the use of pairs of words and word phrases as features
improved performance.

5 Results

This section provides some preliminary empirical evi-
dence that maximum entropy is a competitive text clas-
sification algorithm. The results are based on three dif-
f erent data sets.\(^1\)

5.1 Data Sets and Protocol

The WebKB data set [Craven et al., 1998] contains web
pages gathered from university computer science depart-
ments. The pages are divided into seven categories:
student, faculty, staff, course, project, department and
other. In this paper, we use the four most populous
entity-representing categories: student, faculty, course
and project, all together containing 4199 pages. We did
not use stemming or a stoplist. The resulting vocabulary
has 23830 words.

The Industry Sector hierarchy, made available by Mar-
ket Guide Inc. (www.marketguide.com) consists of com-
pany web pages classified in a hierarchy of industry sec-
tors [McCallum et al., 1998]. There are 6440 web pages
partitioned into the 71 classes that are two levels deep
in the hierarchy. In tokenizing the data we do not stem.
After removing tokens that occur only once or are on a
stoplist, the corpus has a vocabulary of size 29964.

The Newsgroups data set contains about 20,000 arti-
cles evenly divided among 20 UseNet discussion groups
[Joachims, 1997]. Many of the categories fall into con-
fusible clusters: for example, five of them are comp.*
discussion groups, and three of them discuss religion.
When tokenizing this data, we skip the UseNet head-
ers (thereby discarding the subject line); tokens are
formed from contiguous alphabetic characters with no
stemming. Documents containing UU-encoded segments
were discarded. The resulting vocabulary, after remov-
ing words that occur only once or on a stoplist, has 57040
words.

Empirical results with maximum entropy are com-
pared to naive Bayes [Lewis, 1998; Mitchell, 1997], a pop-
ular baseline for text classification. We use the multino-
minal instantiation of naive Bayes [McCallum and Nigam,
1998], which accounts for the number of times each word
occurs. Two variants of multinomial naive Bayes are
tested. In scaled naive Bayes, each word count in a doc-
ument is scaled such that each document has a constant
number of word occurrences. In regular naive Bayes,

\(^1\)These data sets are all available on the Internet. See
Table 2: Classification error (%) of maximum entropy text classification on three data sets, compared to regular and scaled naive Bayes. Each is shown at their optimal vocabulary size, indicated in parentheses. Note that maximum entropy always outperforms regular naive Bayes, but the comparison is mixed with scaled naive Bayes.

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Regular naive Bayes</th>
<th>Scaled naive Bayes</th>
<th>Basic Maximum Entropy</th>
<th>Maximum Entropy w/ Prior</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industry Sector</td>
<td>28.97 (20000)</td>
<td>20.21 (29964)</td>
<td>21.14 (29964)</td>
<td>18.90 (29964)</td>
</tr>
<tr>
<td>NewsGroups</td>
<td>16.15 (57040)</td>
<td>14.43 (57040)</td>
<td>15.77 (57040)</td>
<td>15.14 (57040)</td>
</tr>
</tbody>
</table>

Figure 1: Accuracy over iterations of improved iterative scaling on the Industry Sector dataset with the full vocabulary, where it does best on this dataset. For basic maximum entropy, initially, accuracy is very good, and then degrades slowly, indicating the possibility of overfitting. Problems with overfitting are reduced with a Gaussian prior, and performance improves. Note the scaled vertical axis.

For this reason, 5% of the documents are used in these cases as a validation set for early stopping of IIS iterations. Other maximum entropy experiments run for a fixed number of iterations.

5.2 Experiments

Table 2 shows classification error results for each of the three algorithms on each dataset. The first two columns show performance with the two variations of naive Bayes. As an interesting aside, not that scaled naive Bayes is more accurate than regular naive Bayes on these data sets. The third column shows the performance of basic maximum entropy, without a prior. Note that in all cases, maximum entropy performs better than regular naive Bayes. In some cases, the difference is dramatic; for example on the WebKB dataset, maximum entropy provides a 40% reduction in error over naive Bayes. However, in comparison to scaled naive Bayes, the results are mixed. On WebKB, maximum entropy gives lower error, but for Industry Sector and NewsGroups, it does slightly worse.

On the two datasets where maximum entropy performs worse than scaled naive Bayes, a closer analysis of basic maximum entropy indicates that it is overfitting the training data. The bottom line in Figure 1 shows...
how the accuracy of the maximum entropy classifier progresses on the Industry Sector data through the rounds of improved iterative scaling when a validation set is not used for halting. Note that best performance, 78.8%, is achieved at only the second iteration of IIS. However, by the 20th iteration of IIS, accuracy has declined to 75.8%. Similar trends appear in the 20 Newsgroups data, but not in the WebKB data (Figure 2), where maximum entropy performs better than both naive Bayes variations. These results show that basic maximum entropy is overfitting the data in cases where it is not performing well.

When maximum entropy is used with a Gaussian prior, overfitting is reduced, and performance improves. The top line in Figure 1 shows classification accuracy of maximum entropy when a prior is used. Here, performance does not degrade, and classification error is better. The fourth column of Table 2 shows classification error for these cases. In cases where overfitting was evident, error is decreased. Now, performance on the Industry Sector data set is better than scaled naive Bayes. Performance on the WebKB data set, which had no overfitting problems, is essentially unchanged.

Further analysis indicates some areas for future work. Figure 3 shows error of the classifiers across different vocabulary sizes for WebKB. Here, the error of the maximum entropy classifier increases rather suddenly after several thousand words. This shows that feature selection is an important factor for maximum entropy. In these experiments, feature selection was performed with a method that is natural for naive Bayes. In the next section, we discuss some feature selection techniques that would be more appropriate for maximum entropy.

6 Future Work

Many areas of future work remain. The results indicate that maximum entropy may be sensitive to poor feature selection. Since a feature for maximum entropy is a combination of a class and a word, there is no need to have features for all classes for a vocabulary word. For example, “professor” could be a feature for the faculty and course classes, but not for the student class. An iterative greedy feature selection technique for maximum entropy [Della Pietra et al., 1997] has been shown to create compact representations that result in good maximum entropy performance. We intend to test such an approach for text classification.

In the experiments presented here, the same Gaussian prior variance was used for all feature values. This need not be the case. For features with a large amount of training data, overfitting should not be a problem, and a large variance can be used for the prior. For features with only sparse training data, a strong prior (smaller variance) should be used. Future experiments that adjust the prior based on the amount of training data may improve our results further.

Another area of our ongoing work lies in the representation of features and constraints. In the results presented here, we use scaled counts as features. Preliminary results using unscaled counts indicate that accuracy decreases. We hypothesize that using unscaled counts hurts for long documents where repeated words are given too strong a weight. This suggests using feature functions of the form \( \log(\text{count}) \) or some other sub-linear representation instead of the counts themselves.

One promising aspect of maximum entropy is that it naturally handles overlapping features. For example, we could supplement our word features with bigram, phrase, and even non-text features. Maximum entropy will not be hurt by strong independence assumptions, as would naive Bayes with these features. In future work, we will try to augment maximum entropy with expanded feature classes.

One last area of future work is a more thorough comparison of maximum entropy to other state-of-the-art text classification algorithms on several domains. Ongoing work includes direct comparisons of maximum entropy to support vector machines [Joachims, 1998], k-nearest neighbor [Yang, 1999], and RIPPER [Cohen and Singer, 1996].

In summary, maximum entropy is a technique that is popular for many other natural language tasks. Its overriding principle is one of minimal assumption (maximal entropy) that matches an intuition of how probability distributions should be estimated from data. Empirical analysis shows that maximum entropy is competitive with, and sometimes better than, naive Bayes text classification.

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References

[Berger, 1998]


